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The two-echelon inventory-routing problem with fleet management

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This paper introduces the Two-Echelon Inventory-Routing Problem with Fleet Management. This prob-lem arises under a two-echelon vendor-managed inventory system when a company must make vehicle routing and inventory management decisions, while renting a fleet subject to short- and mid-term agree-ments. Different chemical products are transported contaminating the vehicles that may require cleaning activities. Pickups of input take place in the first echelon, and the final product deliveries are performed in the second echelon. Based on a real-life case in the petrochemical industry, we introduce a formula-tion that takes into account vehicle rentals, cleanings, transportation, inventory management, and vehicle returns decisions. We design a branch-and-cut algorithm to solve it and also propose a matheuristic, in which vehicle routes are handled by an adaptive large neighborhood mechanism, while input pick-ups, product deliveries, and fleet planning are performed by solving several subproblems to optimality. Moreover, we introduce a hybrid parallel framework, combining our matheuristic and the branch-and-cut algorithm in order to solve very large instances exactly. We validate our methods by solving a set of instances of the two-echelon multi-depot inventory-routing problem from the literature, obtaining new best solutions for all instances. We have introduced a set of instances for this rich and new problem, and performed an extensive assessment of our methods. The results provide interesting data about the supply chain structure.

1. Introduction

Modern supply chains require high level of coordination and synchronization of their decisions (Chabot et al., 2018), and these can help decrease logistics costs and improve vendor-customer re-lationships (Yuliang and Dresner, 2008). In this context, Vendor-Managed Inventory (VMI) systems configure a collaborative prac-tice between suppliers and customers Andersson et al. (2010). Un-der the VMI paradigm, suppliers control the inventory of the cus-tomers, deciding when to serve them and how much to deliver. According to Zhao et al. (2010), the VMI practice is mutually ben-eficial, since customers do not have to spend resources to con-trol their inventories and to manage orders, while suppliers im-prove their logistics activities coordination, particularly on delivery routes composition.

To operate a VMI strategy one must solve an Inventory-Routing Problem (IRP), which integrates the inventory management and the multi-period vehicle routing problem into the same framework. Since it was introduced by Bell et al. (1983), a wide range of IRP variants have been proposed, involving strategic, tactical and op-erational criteria (see Coelho et al., 2014). Despite these studies, two simplifications are common in road-based logistics. First, the supply chain is usually simplified into one-echelon only and in-volves one plant serving multiple customers, known as a one-to-many structure (Coelho et al., 2014). Second, fleet decisions are of-ten taken on a tactical perspective, like sizing and mix (Andersson et al., 2010).

Fleet management is one of the most expensive activities in the logistics industry (Shintani et al., 2010). Regarding routing prob-lems, the main concern lies on the strategical and tactical level, with long- and medium-term impacts, as the fleet sizing, when an unlimited number of homogeneous vehicles are considered, or to the fleet composition, when vehicles with different capaci-ties are available to perform deliveries (Bielli et al., 2011), respec-tively. Few studies address operational level issues, such as maintenance, which impacts the short-term vehicle availability, and in the following, we review the relevant literature. Fagerholt and Lindstad (2007) present an interactive optimization-based decision support system for ship routing and scheduling, where a clean-ing procedure can be taken into account when needed. Hvattum et al. (2009) describe a similar approach for a tank allocation prob-lem arising in the shipping of bulk oil and chemical products by tanker ships. In road-based transportation, the majority of stud-ies consider multi-compartment vehicles, as in Oppen et al. (2010), which deal with live animals from farms to slaughterhouses, where vehicle disinfection is performed between consecutive tours. More recently, Lahyani et al. (2015) address a multi-product, multi-period routing problem arising in the collection of olive oil in Tunisia. Due to the difference among olive oil grades, a compart-ment may require a cleaning activity.

While fleet management in an integrated context is a com-plex task, outsourced fleet employment enables companies to fo-cus their efforts on their core competence. According to Windle et al. (1999), the outsourcing of integrated logistics functions al-lows companies to reduce their costs and improve the customer service level.

When more echelons are considered, the integrated sup-ply chain management becomes more complex, and the decision-making process is more difficult. Recently, Guimarães et al. (2019) introduced the two-echelon multi-depot IRP (2E-MDIRP), inspired by a VMI system implementation of a real fuel distribution problem in South America. In that case, plants in the middle layer control the gasoline inventory of a set of gas stations, besides their own inventory of input (ethanol) picked up from supplying facilities. The aim is to minimize pickups and deliveries routing costs, in addition to inventories costs. Despite the costs involved, the authors assume that a fleet of vehicles is available at each plant, omitting any cost for the use and maintenance of the vehicles.

In order to take advantage from the fleet outsourcing, to enable companies to focus on their core business, and to consider a more realistic and integrated scenario, we extend the 2E-MDIRP, incorpo-rating fleet management decisions. We consider a two-echelon (2E) supply chain, in which the plants in the middle layer are responsi-ble for managing inputs pickups from suppliers in the first echelon, and the product deliveries to customers in the second one. Plants operate with a non-compartmentalized outsourced fleet, which must be rented. Cleaning activities must take place whenever the vehicle exchanges the loaded product or when it is returned to the rental company. In addition to traditional transportation and inventory decisions, plants are in charge of fleet planning, includ-ing renting, cleaning, and returning vehicles. These aspects define an unparalleled problem in the literature, which we call the 2E-IRP with fleet management (2E-IRPFM). The aim is to minimize fleet management (rent and cleaning), routing (pickups of the in-put and deliveries of gasoline blend) and inventories (input and final product) costs, avoiding stock-outs over a planning horizon. We also perform extensive computational experiments on a new set of 2E-IRPFM instances, inspired by a real-life case. We design a set of performance indicators and derive many managerial insights for this rich and new problem.

The scientific contributions of this work are:

1. We are the first to consider an IRP while managing fleet deci-sions, not limited to size and mix, but also taking into account rentals, cleanings, and returns. These decisions involve opera-tional tasks related to the outsourced fleet management, allow-ing companies to plan its short-term activities in a more in-tegrated way. Moreover, our approach is more realistic when compared to the classical IRP, since the rental and cleaning de-cisions impact the operational costs, and are frequently ignored in multi-period routing problems;
2. We describe, model, solve, and compare the two classic in-ventory policies (detailed below) with different fleet manage-ment costs. We analyze the performance of these configurations based on their partial costs;
3. We design a branch-and-cut algorithm (B&C) to solve the 2E-IRPFM exactly. As the problem is NP-hard, the B&C is efficient to solve only small instances. In this sense, we also design a matheuristic to provide better results for large instances. Fur-thermore, in order to handle large instances exactly, we em-ploy parallel computing techniques proposing a hybrid exact al-gorithm, combining the matheuristic algorithm with the B&C scheme. This hybridization takes advantages from both original methods, overcoming each one individually;
4. We validate our algorithms on a special case of our prob-lem from the literature, proving optimality for several open in-stances and providing best known solutions to all of them.

The remainder of the paper is organized as follows. In Section 2, we formally describe the 2E-IRPFM, and propose a mixed-integer linear programming (MILP) formulation in Section 3, where we present sets of new and existing valid inequalities. The B&C al-gorithm is detailed in Section 4. In Section 5, we describe the matheuristic algorithm proposed to solve the 2E-IRPFM, while Section 6 presents the hybrid parallel exact approach. In Section 7, we discuss the results of extensive computational experiments per-formed to assess the quality of the algorithms, and we also derive business insights based on the results. Conclusions are presented in Section 8.

2. Problem description

The 2E-IRPFM is defined over an undirected graph $G = (V, E)$, where the vertex set V represents the union of the sets F of suppliers, P of plants, and C of customers, while E is the set of edges. The first echelon links suppliers and plants and it is defined by subgraph $G = (V, E)$, where $V = F \cup P$ and $E = (u, v) : u, v \in V, u \in F \wedge v \in P$. The second echelon links plants and customers and is defined by subgraph G , with $V = P \cup C$ and $E = (u, v) : u, v \in V \wedge u, vP, u < v$, in which $V = V \cup V$ and $E = E \cup E$. A non-negative cost c_{uv} is associated with each edge $(u, v) \in E$.

The planning horizon is defined over a set $T = \{1, . . . , p\}$ of pe-riods. In each period t, each plant $j \in P$ is allowed to rent up to $|K|$ of homogeneous vehicles of capacity Q at rental cost f_W per vehicle per period. Each vehicle can be used to pick up a certain amount of input (α) from a supplier, and/or to deliver a certain amount of final product (β) to customers. Once a customer is visited and one must determine the quantity to be delivered, one of two policies are often applied. Under the maximum level (ML) policy, the plant is free to decide how much to deliver to a customer, as long as the inventory capacity is not exceeded. The order-up-to (OU) policy fills the customer's inventory capacity whenever a delivery occurs (Archetti et al., 2007).

After performing a pickup of α (delivery of β), the vehicle re-mains contaminated with α (β). Before a new trip with a different product, each vehicle must undergo a chemical cleaning procedure, incurring in cleaning cost f_S. This can occur in the same period, if α is picked up at the beginning of t, and β is delivered at the end of t by the same vehicle k, or in different periods, if the vehicle re-mains at the plant. In this case, f_W is due for each additional period in which the vehicle remains at the plant. A vehicle contaminated with α in t can perform a new pickup in t , $t > t$ without going through chemical cleaning, since it does not perform any delivery of β between [t, t]. The same idea works for β. As a vehicle remains contaminated with the last load, when a plant decides to return it, a cleaning cost f_S is due as every vehicle must be cleaned before being returned.

Each unit of β requires a certain quantity ϕ of α in its blend, and the demand d_l^t of each customer l in each period t is known a priori. Plant j has a maximum inventory capacity U_j for α, and its level cannot be lower than L_j. One unit of α incurs an inventory holding cost h_j per period. Likewise, U_l and L_l denote the maxi-mum and minimum inventory levels for β at customer l, with unit holding cost h_l. The total availability of α at all suppliers is not constrained. However, each supplier i disposes of $_i$ units for the whole planning horizon, according to a pre-established contract with plants. In $t = 0$, there are no rented vehicles at the plants, and initial inventory levels I^0_j and I_l^0 are known at each plant j and each customer l.

Regarding the timing of the activities, we assume that α is al-ways picked up at the beginning of a period if a pickup is needed. When a delivery is also required, it must be scheduled after all the pickups had been performed. That is mandatory to enable the blending process and to produce β in the same period. A vehicle must be clean before its return to the rental company at the end of a period.

We justify the sequencing of these decision based on real data from our partner, within a daily journey of 14 h. From 5 to 6 a.m. the vehicle is cleaned, when it remained contaminated at plant, or it is taken from the rental company. Pickups of α occur from 6 to 9 a.m. Unloading of α, the blending process and the vehicle loading with β is performed from 9 to 11 a.m. Since the vehicle cleaning is an independent task, its can be done after the unloading, while α and β are being blended. Deliveries routes last around 6 h, starting at 11 and ending at 17 h. If the vehicle is returned in the same period, it is cleaned from 17 to 18 h, and returned before 19 h to the rental company.

The objective of the 2E-IRPFM is to minimize the total inven-tory, transportation, and fleet management cost, determining, for each plant:

- When, how much and from which supplier to pickup α;
- When and how much to deliver β to a customer;
- When and how many vehicles to rent, to clean, to keep, and to return;
- How to combine customer deliveries into vehicle routes.

A customer may be visited at most by one vehicle per pe-riod. Likewise, a plant is allowed to pick up α from one sup-plier using one vehicle in a period. Each input pickup or delivery route must end at the same starting plant. As the fleet is non-compartmentalized, a pickup and a delivery cannot be combined in the same tour. Thus, after collecting α from a supplier, the ve-hicle needs to return to its original plant and will only be able to perform a delivery route if it is properly cleaned.

3. Mathematical formulation for the 2E-IRPFM

We now present the mathematical formulation for the 2E-IRPFM. For each plant j one must determine the quantity of prod-uct q^{kt}_{jl} delivered to customer l and the total amount of input $r_i{}^{kt}{}_j$ picked up from supplier i, using vehicle k in period t. At the end of each period, the inventory level of β at customer l is given by I_l^t, while the inventory level of α at plant j is I^t_j. The remaining variables used in our model are:

- $W_j^{kt} = 1$ if vehicle k is rented by plant j in period t, 0 otherwise;
- $R^{kt}_j = 1$ if vehicle k rented by plant j is returned in period t, 0 otherwise;
- $X_i{}^{kt}{}_j = 1$ if vehicle k rented by plant j picks up α from supplier i in period t, 0 otherwise;
- $Y_{jl}^{kt} = 1$ if vehicle k rented by plant j delivers β to customer l in period t, 0 otherwise;
- $y^k_{uv}{}^{jt} = 1$ if vehicle k rented by plant j travels between customers u and v, $u < v$, in period t, 0 otherwise;
- $y^{k}_{jl}{}^{jt} \in \{0, 1, 2\}$. When $y^{k}_{jl}{}^{jt} = 1$, vehicle k rented by plant j trav-els directly from plant j to customer l in period t. If $y^{k}_{jl}{}^{jt} = 2$, a round trip is defined, 0 otherwise;
- $Z^{\alpha}{}_j{}^{,kt} = 1$ if vehicle k rented by plant j ends period t contami-nated with α, 0 otherwise;
- $Z^{\beta}{}_j{}^{,kt} = 1$ if vehicle k rented by plant j ends period t contami-nated with β, 0 otherwise;
- $S^{\alpha}{}_j{}^{,kt} = 1$ if vehicle k rented by plant j contaminated with β is cleaned to pickup α in period t, 0 otherwise;
- $S^{\beta}{}_j{}^{,kt} = 1$ if vehicle k rented by plant j contaminated with α is cleaned to deliver β in period t, 0 otherwise;
- $S^{R}{}_j{}^{,kt} = 1$ if vehicle k rented by plant j, contaminated with α or β, is cleaned to be returned in period t, 0 otherwise;
- S^{kt}_j : total number of cleanings of vehicle k rented by plant j in period t.

In addition, we also define variable $X_j^{kt} = 1$ to indicate that ve-hicle k, rented by plant j, performs a pickup in t, 0 otherwise. Equivalently, $Y_j^{kt} = 1$ works for a delivery, 0 otherwise. Without loss of generality, we assume that no vehicle is housed at plants in $t = 0$, i.e., W_j^{k0}, R^{k0}_j, $Z^{\alpha}{}_j{}^{,k0}$ and $Z^{\beta}{}_j{}^{,k0}$ are set to zero.

A suitable representation of the 2E-IRPFM is shown on Fig. 1. Considering two consecutive periods, t and $t + 1$, we depict the sequence of decisions for a given vehicle k, rented by plant j. In order to simplify the example, indices k and j are omitted on the variables. First, the vehicle is rented in period t, which obviously implies in $W^t = 1$. Then, this vehicle performs a pickup, becom-ing contaminated with α. Before it can be assigned to a delivery, a cleaning is carried out, i.e., $S^{\beta,t} = 1$. The vehicle then leaves to the plant contaminated with β, and remains there in the following period, yielding $Z^{\beta,t} = 1$ and an additional lease with $W^{t+1} = 1$. At the beginning of $t + 1$, the vehicle is cleaned to perform a new in-put pickup ($S^{\alpha,t+1} = 1$). After returning to the plant, one last clean-ing is performed ($S^{R,t+1} = 1$), and the vehicle is returned at the end of $t + 1$.

The 2E-IRPFM is formulated by (1)–(42).

$$\min \sum_{t\in T}\Big(\sum_{j\in P}\sum_{k\in K} f_w W_j^{kt} + \sum_{j\in P}\sum_{k\in K} f_S S^{kt}_j + \sum_{j\in P} h_j I^t_j + \sum_{l\in C} h_l I_l^t + \sum_{k\in K}\sum_{(i,j)\in E} 2c_{ij} X_i{}^{kt}{}_j + \sum_{j\in P}\sum_{k\in K}\sum_{(u,v)\in E} c_{uv} y_{uv}^{k\,jt}\Big) \tag{1}$$

subject to

$$\sum_{t\in T}\sum_{j\in P}\sum_{k\in K} r_i{}^{kt}{}_j \leq {}_i \qquad i \in F \tag{2}$$

$$I^t_j = I^{t-1}_j + \sum_{i\in F}\sum_{k\in K} r^{kt}_{ij} - \sum_{l\in C}\sum_{k\in K} \phi q^{kt}_{jl} \qquad j \in P,\ t \in T \tag{3}$$

$$I^t_l = I^{t-1}_l + \sum_{j\in P}\sum_{k\in K} q^{kt}_{jl} - d^t_l \qquad l \in C,\ t \in T \tag{4}$$

$$L_l \leq I_l^t \leq U_l \qquad l \in V,\ t \in T \tag{5}$$

$$\sum_{k\in K}\sum_{i\in F} X_i{}^{kt}{}_j \leq 1 \qquad j \in P,\ t \in T \tag{6}$$

$$\sum_{i\in F}\sum_{k\in K} r^{kt}_{ij} \leq U_j - I^{t-1}_j \qquad j \in P,\ t \in T \tag{7}$$

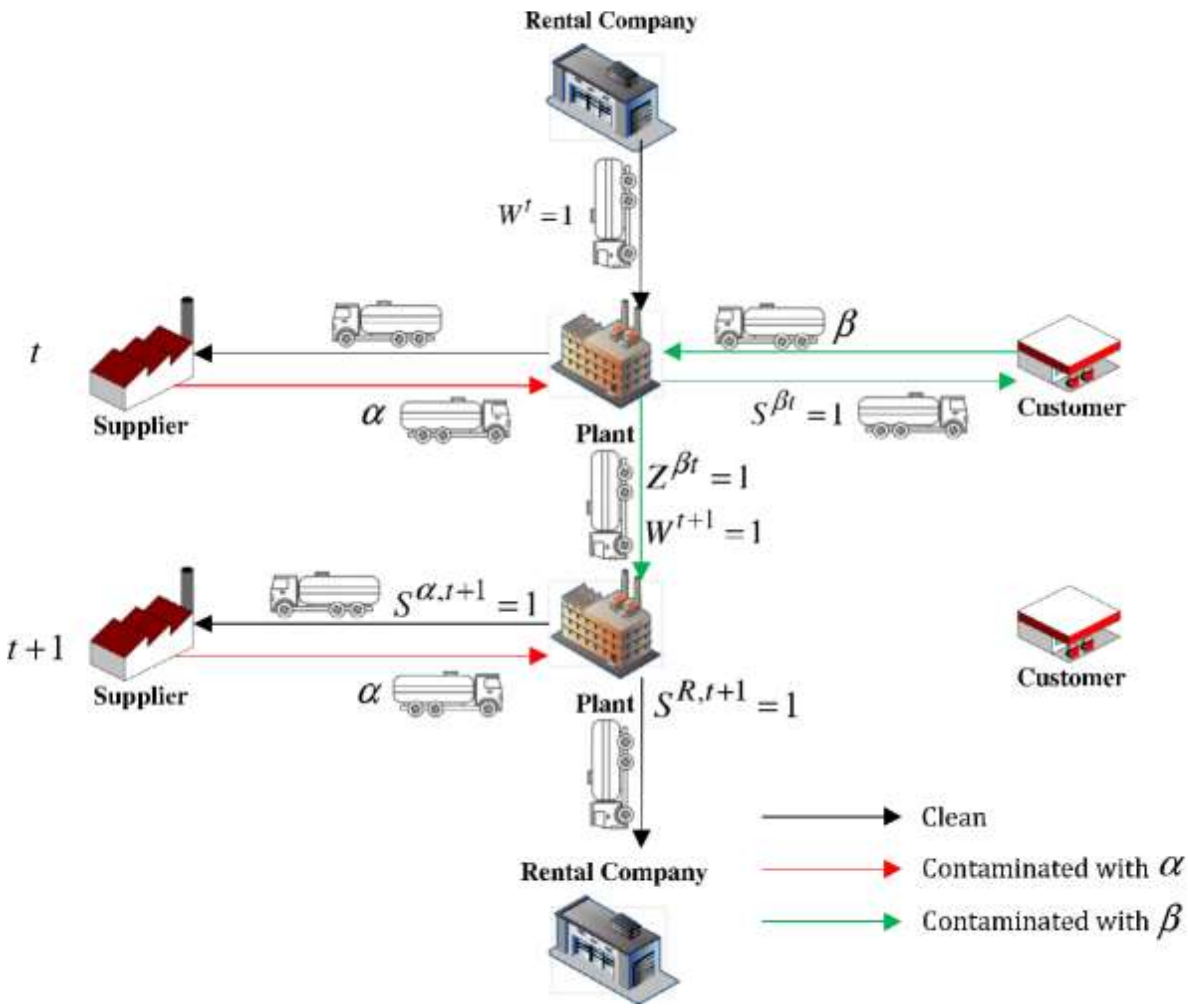

Fig. 1. Graphical representation of the 2E-IRPFM.

$$r_{ij}^{kt} \le Q X_{ij}^{kt} \quad j\in P, i\in F, k\in K, t\in T \tag{8}$$

$$\sum_{j\in P}\sum_{k\in K} q_{jl}^{kt} \le U_l - I_l^{t-1} \quad l\in C, t\in T \tag{9}$$

$$q_{jl}^{kt} \le U_l Y_{jl}^{kt} \quad l\in C, j\in P, k\in K, t\in T \tag{10}$$

$$\sum_{l\in C} q_{jl}^{kt} \le Q Y_{jj}^{kt} \quad j\in P, k\in K, t\in T \tag{11}$$

$$\sum_{j\in P}\sum_{k\in K} Y_{jl}^{kt} \le 1 \quad l\in C, t\in T \tag{12}$$

$$\sum_{u\in V} y_{uv}^{kjt} + \sum_{u\in V} y_{vu}^{kjt} = 2Y_{jv}^{kt} \quad v\in V, j\in P, k\in K, t\in T \tag{13}$$

$$\sum_{l\in S}\sum_{u\in S, l<u} y_{lu}^{kjt} \le \sum_{l\in S} Y_{jl}^{kt} - Y_{jm}^{kt} \quad S\subseteq C, |S|\ge 2, m\in S, j\in P, k\in K, t\in T \tag{14}$$

$$\sum_{i\in F} X_{ij}^{kt} = X_{jj}^{kt} \quad j\in P, k\in K, t\in T \tag{15}$$

$$X_{jj}^{kt} \le W_j^{kt} \quad j\in P, k\in K, t\in T \tag{16}$$

$$Y_{jj}^{kt} \le W_j^{kt} \quad j\in P, k\in K, t\in T \tag{17}$$

$$R_j^{kt} \le W_j^{kt} \quad j\in P, k\in K, t\in T \tag{18}$$

$$Z_j^{\alpha,k,t-1} \le W_j^{kt} \quad j\in P, k\in K, t\in T \tag{19}$$

$$Z_j^{\beta,k,t-1} \le W_j^{kt} \quad j\in P, k\in K, t\in T \tag{20}$$

$$Z_j^{\alpha,kt} + Z_j^{\beta,kt} + R_j^{kt} = W_j^{kt} \quad j\in P, k\in K, t\in T \tag{21}$$

$$X_{jj}^{kt} - Y_{jj}^{kt} - R_j^{kt} \le Z_j^{\alpha,kt} \quad j\in P, k\in K, t\in T \tag{22}$$

$$Z_j^{\alpha,k,t-1} - Z_j^{\beta,kt} - R_j^{kt} \le Z_j^{\alpha,kt} \quad j\in P, k\in K, t\in T \tag{23}$$

$$Y_{jj}^{kt} - R_j^{kt} \le Z_j^{\beta,kt} \quad j\in P, k\in K, t\in T \tag{24}$$

$$Z_j^{\beta,k,t-1} - Z_j^{\alpha,kt} - R_j^{kt} \le Z_j^{\beta,kt} \quad j\in P, k\in K, t\in T \tag{25}$$

$$W_j^{k,t-1} - R_j^{k,t-1} \le W_j^{kt} \quad j\in P, k\in K, t\in T \tag{26}$$

$$X_{jj}^{kt} + Z_j^{\beta,k,t-1} - 1 \le S_j^{\alpha,kt} \quad j\in P, k\in K, t\in T \tag{27}$$

$$Z^{\beta}_{j,k,t-1} + Z^{\alpha}_{j,kt} - 1 \le S^{\alpha}_{j,kt} \quad j \in P, k \in K, t \in T \tag{28}$$

$$Y_{jj}^{kt} + X_{jj}^{kt} - 1 \le S^{\beta,kt}_j \quad j \in P, k \in K, t \in T \tag{29}$$

$$Y^{kt}_{jj} + Z^{\alpha,k,t-1}_j - 1 \le S^{\beta,kt}_j \quad j \in P, k \in K, t \in T \tag{30}$$

$$Z^{\alpha,k,t-1}_j + Z^{\beta,kt}_j - 1 \le S^{\beta,kt}_j \quad j \in P, k \in K, t \in T \tag{31}$$

$$R^{kt}_j \le S^{R,kt}_j \quad j \in P, k \in K, t \in T \tag{32}$$

$$S^{\alpha,kt}_j + S^{\beta,kt}_j + S^{R,kt}_j = S^{kt}_j \quad j \in P, k \in K, t \in T \tag{33}$$

$$I^t_j, r^{kt}_{ij} \ge 0 \quad i \in F, j \in P, k \in K, t \in T \tag{34}$$

$$I^t_l, q^{kt}_{jl} \ge 0 \quad l \in C, j \in P, k \in K, t \in T \tag{35}$$

$$Y^{kt}_{jj}, S^{R,kt}_j, S^{\beta,kt}_j, S^{\alpha,kt}_j, R^{kt}_j, Z^{\beta,kt}_j, Z^{\alpha,kt}_j, W^{kt}_j \in \{0,1\} \quad j \in P, k \in K, t \in T \tag{36}$$

$$S^{kt}_j \in \{0,1,2,3\} \quad j \in P, k \in K, t \in T \tag{37}$$

$$X^{kt}_{ij} \in \{0,1\} \quad i \in F, j \in P, k \in K, t \in T \tag{38}$$

$$X^{kt}_{jj} \in \{0,1\} \quad j \in P, k \in K, t \in T \tag{39}$$

$$Y^{kt}_{jl} \in \{0,1\} \quad l \in C, j \in P, k \in K, t \in T \tag{40}$$

$$y^{kjt}_{jl} \in \{0,1,2\} \quad l \in C, j \in P, k \in K, t \in T \tag{41}$$

$$y^{kjt}_{uv} \in \{0,1\} \quad u, v \in C, u < v, j \in P, k \in K, t \in T. \tag{42}$$

The objective function (1) minimizes the total cost, given by six terms: fleet rental, fleet cleaning, input inventory at the plants, final product inventory at the customers, pickup transportation, and delivery transportation costs. Constraints (2) bound the in-put availability, according to the contract between suppliers and plants. Constraints (3)–(5) balance the flow and impose inventory bounds. Constraints (6) allow at most one pickup per plant per period, while constraints (7) ensure the ML inventory policy for

α. Constraints (8) guarantee that the vehicle capacity is not ex-ceeded. The ML inventory policy for β is formulated by (9). Con-straints (10) link assignment variables with the quantity delivered. Constraints (11) guarantee that total delivery quantity does not ex-ceed the vehicle capacity, while (12) avoid split deliveries. Con-straints (13) ensure the linking conditions: when a node l is vis-ited, it should have exactly one incoming edge and one outgoing edge. In particular, when a vehicle route contains only one cus-

tomer, obviously visited in a round trip, only one routing vari-able $y^{kjt}_{jl} = 2$ is sufficient to represent both incoming and outgo-
ing edges, reducing the total number of decision variables in the model. Constraints (14) eliminate subtours, by requiring for each proper and nonempty subset S of C, the total number of edges traveled by vehicle k from plant j between the nodes of S must be at most $|S| - 1$, while constraints (15) link a pickup with the as-signed vehicle. Constraints (16)–(33) formulate the fleet rental and return. In particular, constraints (16) and (17) require a vehicle to be rented if it is used to either pickup α or deliver β. Constraints

(18) impose that only rented vehicles can be returned. Constraints (19) and (20) impose that if a vehicle remains contaminated at the plant, it must be rented by one more period. Constraints (21) de-fine mutually exclusive conditions for a vehicle at the end of t:

contaminated with α, contaminated with β, or returned to the rental company. In the last case, the vehicle must be cleaned. Con-
straints (22) establish the first condition for a vehicle that remains at the plant contaminated with α, which occurs when it performs a pickup, and does not perform a delivery and/or is returned in
the same period. The second condition is assured by constraints (23), in which a vehicle finishes contaminated with α in period

$t-1$ and remains at the plant without performing a delivery in t. Analogously, a vehicle remains contaminated with β if it performs a delivery in period t and does not return to the rental company
in the same period, which is imposed by constraints (24). Besides that, constraints (25) describe the second condition for a contami-
nation with β, that occurs when the vehicle remains contaminated from previous period $(t-1)$, neither performing a pickup or being returned to the rental company in t. Likewise, constraints (26) en-
sure vehicle flow conservation. Constraints (27) and (28) encom-pass all the requirements to clean up the vehicle, in order to allow a pickup. In particular, a vehicle must be cleaned if it was contam-inated with β in $t-1$, and it is scheduled to pickup α in t, which
is handled by (27). Another situation occurs whenever a vehicle finishes contaminated with different products in two consecutive periods (β in $t-1$ and α in t). In this case, constraints (28) impose
a clean up in order to enable a pickup in t. Constraints (29) ensure a clean up whenever the same vehicle performs a pickup and a
delivery in t, while (30) consider a similar case, when the vehicle
is already contaminated with α in $t-1$ and carry out a delivery
of β in t. Constraints (31) work for β as (28) work for α. Con-
straints (32) ensure that the vehicle is clean when returned, while constraints (33) compute the total number of cleanings. Lastly, con-
straints (34)–(42) define the variables domain.

The order-up-to level inventory policy (OU) links the decision of *when* and *how much* to serve a customer. Thereby, whenever

a plant performs a delivery, the total quantity delivered must be equal to the customer inventory availability:

$$\sum_{j \in P}\sum_{k \in K} q^{kt}_{jl} \ge U_l \sum_{j \in P}\sum_{k \in K} Y^{kt}_{jl} - I^{t-1}_l \quad l \in C, t \in T. \tag{43}$$

3.1. Valid inequalities

In order to strengthen the formulation and improve the qual-ity of its dual bound, we also present a set of well-known valid inequalities from basic IRPs (Archetti et al., 2007; Bertazzi et al., 2019; Coelho and Laporte, 2014). These are described next.

$$y^{kjt}_{jl} \le 2Y^{kt}_{jl} \quad l \in C, j \in P, k \in K, t \in T \tag{44}$$

$$y^{kjt}_{ul} \le Y^{kt}_{jl} \quad u, l \in C, u < l, j \in P, k \in K, t \in T \tag{45}$$

$$y^{kjt}_{lu} \le Y^{kt}_{jl} \quad u, l \in C, l < u, j \in P, k \in K, t \in T \tag{46}$$

$$Y^{kt}_{jl} \le Y^{kt}_{jj} \quad l \in C, j \in P, k \in K, t \in T. \tag{47}$$

Inequalities (44) strengthen the case if customer l is served in a direct delivery by vehicle k rented by plant j in period t. Sim-ilarly, inequalities (45) and (46) work for customer l if it is pre-ceded or succeeded by another customer u, respectively. Inequali-ties (47) ensure that customer l is served by vehicle k starting from plant j in period t, only if the vehicle is assigned for that plant.

We also consider inequalities (48)–(50) adapted by Guimarães et al. (2019) for the multi-depot multi-vehicle IRP, which is useful to determine the minimum number of deliveries to avoid stock-out at customer l on the range $[t_1, t_2]$. Particularly, the right-hand side (RHS) of inequalities (48) is a delivery lower bound. If the demand during $[t_1, t_2]$ is greater than the maximum possible inventory held, regardless of the current inventory level, the customer must be served in $[t_1, t_2]$. The minimum number of deliveries depends on the capacities Q and U_l. Suppose $[t_1 = 1, t_2 = 4]$, a customer l with $d_l^1 = d_l^2 = d_l^3 = d_l^4 = 50$, $I_l^0 = 50$, $U_l = 150$ and $Q = 100$. The RHS of (48) is $\frac{200-100}{min\{100,150\}} = 1$, thus at least one delivery must be performed to be tightened by considering the current inventory level instead of the customer capacity, according to the RHS of (49). In this case, the numerator contains a decision variable and because of that, the RHS cannot be rounded up. Based on the previous example, the RHS of (48) is $\frac{200-50}{min\{100,150\}} = 1.5$, which is stronger than before.

As pointed by Coelho and Laporte (2014), these inequalities can be written in a different way. If the inventory held at the beginning of the interval $[t_1, t_2]$ is sufficient to fulfill its accumulated demand, then no visit to customer l is required, i.e., if $\sum_{t=t_1}^{t_2} d_l^t \ge I_l^{t_1-1}$, then $\sum_{j\in P}\sum_{k\in K}\sum_{t=t_1}^{t_2} Y_{jl}^{kt} \ge 1$. On the other hand, if the inventory is not sufficient to meet future demands, then a visit must take place, as shown by inequalities (50). Guimarães et al. (2019) also pro-posed inequalities (51), which identify, based on the same idea, the smallest range $[t_1, t_2]$ for which a plant j must perform a pickup.

$$\sum_{j\in P}\sum_{k\in K}\sum_{t=t_1}^{t_2} Y_{jl}^{kt} \ge \frac{\sum_{t=t_1}^{t_2} d_l^t - U_l}{\min\{Q, U_l\}} \qquad l \in C,\ t_1, t_2 \in T,\ t_2 > t_1 \tag{48}$$

$$\sum_{j\in P}\sum_{k\in K}\sum_{t=t_1}^{t_2} Y_{jl}^{kt} \ge \frac{\sum_{t=t_1}^{t_2} d_l^t - I_l^{t_1-1}}{\min\{Q, U_l\}} \qquad l \in C,\ t_1, t_2 \in T,\ t_2 > t_1 \tag{49}$$

$$\sum_{j\in P}\sum_{k\in K}\sum_{t=t_1}^{t_2} Y_{jl}^{kt} \ge \frac{\sum_{t=t_1}^{t_2} d_l^t - I_l^{t_1-1}}{\sum_{t=t_1}^{t_2} d_l^t} \qquad l \in C,\ t_1, t_2 \in T,\ t_2 > t_1 \tag{50}$$

$$\sum_{i\in F}\sum_{k\in K}\sum_{t=t_1}^{t_2} X_{ij}^{kt} \ge \frac{\sum_{l\in C}\sum_{k\in K}\sum_{t=t_1}^{t_2} \phi q_{jl}^{kt} - I_j^{t_1-1}}{\min\{Q, U_j\}} \qquad j \in P,\ t_1, t_2 \in T,\ t_2 > t_1 \tag{51}$$

Based on the features of the 2E-IRPFM, we introduce the fol-lowing new inequalities.

$$\frac{\sum_{l\in C}\sum_{k\in K} q_{jl}^{kt}}{Q} \le \sum_{k\in K} W_j^{kt} \qquad j \in P,\ t \in T \tag{52}$$

$$\sum_{k\in K} Y_j^{kt} \le \sum_{k\in K} W_j^{kt} \qquad j \in P,\ t \in T \tag{53}$$

$$Y_j^{kt} + X_j^{kt} \le W_j^{kt} + S_j^{\beta,kt} \qquad j \in P,\ k \in K,\ t \in T. \tag{54}$$

The left hand side of (52) computes the minimum number of vehicles required, based on the total delivery amount scheduled in t. Thus, the right hand side sets the lower bound for the number of rented vehicles per plant. Equivalently, this lower bound is also obtained according to the number of vehicles scheduled for deliv-eries, departing from each plant, as presented by inequalities (53). Finally, inequalities (54) ensure that if vehicle k, housed at plant j, is assigned to a pick up and a delivery in period t, then this vehicle must be rented and cleaned.

It is known that the first-in, first-out rule can be associated with an optimal solution of the IRP. Desaulniers et al. (2015) then introduce the following notation. Let $I_l^{0,\zeta} = \max\{0, I_l^0 - \sum_{t=1}^{\zeta} d_l^t\}$ be the remaining quantity of initial inventory for customer l at the end of period $\zeta \in T$. The residual demands $(\bar{d}_l^{\zeta})$ define the demand not met by the initial inventory:

$$\bar{d}_l^{\zeta} = \begin{cases} \max\{0, d_l^{\zeta} - I_l^{0,\zeta-1}\} & \text{otherwise } \forall \zeta \in T. \\ \max\{0, d_l^1 - I_l^0\} & \text{if } \zeta = 1 \end{cases} \tag{55}$$

In addition, the same authors combine the maximum inventory level (U_l), the demand (d_l^{ζ}) and the residual demand $(\bar{d}_l^{\zeta})$ in $\zeta \in T$ for each customer l, defining the following set P_{lt}^+, containing all periods in which a sub-delivery of β for a customer l in period t can be used, either to satisfy the current demand or to be held in inventory for future periods.

$$P_{lt}^+ = \left\{\zeta \ge t : \bar{d}_l^{t} > 0\right\} \cup \left\{\zeta > t : \bar{d}_l^{\zeta} > 0 \text{ and } \sum_{t'=t}^{\zeta-1} d_l^{t'} < U_l\right\} \cup \left\{\zeta : \sum d_l^t < U_l\right\}. \tag{56}$$

Finally, let $P_{l\zeta}^- = \{t \in T \mid \zeta \in P_{lt}^+\}$ be the set of periods for which a delivery can be $\zeta \in T$. Desaulniers et al. (2015) propose the following valid inequalities for the IRP, derived from the minimum number of sub-deliveries per demand. We adapt them to the 2E-IRPFM, as follows:

$$\sum_{j\in P}\sum_{k\in K}\sum_{t\in P_{l\zeta}^-} Y_{jl}^{kt} \ge 1, \qquad l \in C,\ \zeta \in T, \quad \text{with} \quad P_{l\zeta}^- = \emptyset. \tag{57}$$

Lefever (2018) propose a set of valid inequalities to the IRP with transshipment (IRPT), useful to bound the minimum number of delivery routes along the planning horizon T. We adapt these in-equalities to the 2E-IRPFM, as:

$$\sum_{j\in P}\sum_{k\in K}\sum_{t=1}^{\zeta} Y_{jl}^{kt} \ge \frac{\sum \bar{d}^{\zeta}}{Q} \qquad \zeta \in T. \tag{58}$$

Finally, to strengthen the IRP inventory management compo-nent formulation, Lefever et al. (2018) employ a remaining quan-tity to restrict the range of continuous variables I_l^t and q_{jl}^{kt}. These improved bounds can be tightened as follows.

$$I_l^t \ge I_l^{0,t}, \qquad l \in C,\ t \in T \tag{59}$$

$$q_{jl}^{kt} \le U_l - I_l^{0,t}, \qquad j \in P,\ l \in C,\ k \in K,\ t \in T. \tag{60}$$

4. Branch-and-cut algorithm

Due to their combinatorial features, the number of subtour elimination constraints (SEC) (14) is too large, and their full enu-meration is impracticable. To overcome this limitation, these con-straints must be dynamically generated along the search process. We use an exact approach, known as branch-and-cut, to solve the model presented in Section 3, where SEC are added to the search tree whenever subtours are found at the current solution.

At the beginning of the search, all valid inequalities are gen-erated and added in the root node. Whenever a node of the search tree is solved by a MIP solver, a search for violated SEC is performed. We have used the CVRPSEP package of Lysgaard et al. (2004) to generate SEC. When subtours are identified by CVRPSEP, their corresponding SEC are added to the search tree. This process is repeated until a feasible or dominated solution is found, or until there are no more cuts to be added. At this point, a new subproblem is generated by branching on a fractional vari-able, and the model is reoptimized in a new node. We provide a scheme of our branch-and-cut algorithm on Algorithm 1.

Algorithm 1 Pseudocode of the proposed B&C algorithm.

```
At the root node of the search tree, generate (1)– (13), (15)–(42) and all valid inequalities (44)–(54), (57)–(60).
Solve the linear problem (LP) relaxation of the node.
Termination check:
if there are no more nodes to evaluate then
    Stop.
else
    Select one node from the B&C tree.
end if
while the solution of the current LP relaxation contains subtours do
    Add violated subtour elimination constraints.
    Solve the LP relaxation of the node.
end while
if the solution of the current LP relaxation is in-teger then
    Go to the termination check.
else
    Branch on one of the fractional variables.
    Go to the termination check.
end if
```

5. Matheuristic-based adaptive large neighborhood search algorithm

Since the 2E-IRPFM generalizes the vehicle routing problem (VRP), it is NP-hard and traditional exact methods, like B&C al-gorithms, can solve only small size instances. To handle large in-stances, we propose a matheuristic approach, combining math-ematical programming techniques with heuristic search proce-dures. Matheuristics are widely used to solve routing prob-lems (see Archetti and Speranza, 2014). In particular, Archetti et al. (2017) combine integer programming with tabu search to solve a multi-vehicle IRP. More recently, Bertazzi et al. (2019) pro-pose a matheuristic to solve a multi-depot IRP.

In the context of the 2E-IRPFM we make use of an adaptive large neighborhood search (ALNS) mechanism, responsible for han-dling the delivery routes, while pickup, delivery quantities, fleet management, and some improvements are determined exactly by mixed integer programming (MIP) subproblems.

5.1. ALNS mechanism

The ALNS was proposed by Pisinger and Ropke (2007) to solve the VRP and its extensions. In the inventory-routing context, it has since been successfully applied to several variants (Aksen et al., 2014; Coelho et al., 2012a; Coelho et al., 2012b; Guimarães et al., 2019). Our ALNS deals with the delivery routes, while de-livery quantities, pickups and fleet decisions are determined ex-actly by MIP subproblems. The search procedure follows the gen-eral scheme proposed by Pisinger and Ropke (2007) and is divided into segments. Given a solution, some customers are either in-serted, removed, or swapped among routes at each iteration, by specific operators. Each operator i contains three attributes. The first one is the weight, given by ω_i, whose value depends on past performance. The second attribute is the score, given by π_i, which quantifies the effect on the solution when the operator is applied. It increases by σ_1 if the operator leads to a new best solution, by

σ_2 if it finds a solution better than the current one, and by σ_3 if the solution is worse but is still accepted according to a simulated annealing criteria; the third and last attribute is ς_{ij}, which mea-sures the number of times the operator i has been applied in the last segment $j \in$.

Following the original ALNS framework, we adopt a simulated annealing acceptance criterion. Let s be a solution and s a neigh-bor solution, obtained from s. The acceptance probability of s is $e^{(z(s)-z(s))/\tau}$, where $z(\cdot)$ is the solution cost and $\tau > 0$ is the current temperature. The initial temperature is τ_{start}, decreasing at each iteration by a cooling rate factor φ, with $0 < \varphi < 1$.

In the first iteration, all scores are equal to zero, and all weights are equal to one. A roulette wheel mechanism controls the choice of operators. Given h operators, operator i is chosen with proba-bility $\omega_i / \sum_{j=1}^{h} \omega_j$. In each segment , if the operator is chosen, a reaction factor $\eta \in$ [0, 1] is applied to balance its weight be-tween the past and present performance, according to (61). After this step, all scores are reset to zero.

$$\omega_i := \begin{cases} (1-\eta)\omega_i + \eta\pi_i/\varsigma_{ij} & \text{if } \varsigma_{ij} = 0. \\ \omega_i & \text{if } \varsigma_{ij} = 0 \end{cases} \quad (61)$$

A neighbor solution s is obtained when an operator is selected and applied on s. In the list below, we present the operators de-veloped for our ALNS.

1. ***Randomly remove*** ρ: It randomly selects a period t, a plant j, a vehicle k rented by this plant, and a customer l served from it and removes this customer. It is repeated ρ times.
2. ***Remove worst*** ρ: This operator computes the transportation saving of not visiting each client served, according to the tri-angle inequality. It is applied ρ times, removing, at each time, the customer yielding the highest saving.
3. ***Shaw removal route based***: It randomly selects a period t, a plant j, a vehicle k rented by this plant, and a customer l served by this vehicle. It then computes the distance $min(c_{lu})$ to the closest customer u also being served by the same vehicle, and removes all customers within 2 $min(c_{lu})$ from customer l.
4. ***Empty one period***: Randomly selects a period t and removes all its deliveries.
5. ***Empty one vehicle***: Randomly selects a rented vehicle k and re-moves all deliveries from all plants in all periods performed by this vehicle.
6. ***Empty one plant***: Randomly selects a plant j and removes all deliveries of all vehicles from this plant over the planning hori-zon.
7. ***Farthest customer***: This operator randomly selects a period t, a plant j, and a vehicle rented by this plant, and removes its far-thest customer, measured by the direct distance from the depot. It is repeated ρ times.
8. ***Avoid consecutive visits***: Starting from $t = 1$, it removes the second visit of all customers receiving deliveries in two consec-utive periods. The delivery quantities are updated at each new period analyzed.
9. ***Remove*** ρ ***minimum residual deliveries***: This operator calcu-lates the residual delivery for all served customers, given by $\frac{q_{jl}^{kt}}{U_l - I_l^{t-1}}$, with $q_{jl}^{kt} > 0$, for each $l \in C$, and removes the customer with the minimum residual delivery. It is repeated ρ times.
10. ***Remove*** ρ ***most visited customers***: This operator removes the most visited customer over the planning horizon. Ties are bro-ken randomly. It is repeated ρ times.
11. ***Randomly insert*** ρ: Randomly selects a period t, a plant j and a vehicle k rented by this plant, and inserts a randomly selected customer l unserved in t, following the cheapest insertion rule. The operator is repeated ρ times.
12. ***Insert best*** ρ: Similar to ***Remove worst*** ρ, this operator inserts the customer with the smallest increase in transportation cost. It is repeated ρ times.

13. ***Assignment to the nearest plant***: This operator randomly se-lects a period t and a customer l not served in that period, and inserts it on the route of its nearest plant, following the cheap-est insertion rule. It is repeated ρ times.
14. ***Shaw insertion***: This operator randomly selects a period t and a customer l not served in t. Then it is computes $min(c_{lv})$, with $v \in V$. All customers not yet served in that period, distant up to 2 $min(c_{lv})$ are inserted in a rented vehicle k in t, following cheapest insertion rule. It is repeated ρ times.
15. ***Swap ρ customers***: Randomly selects two customers served by different vehicles and swaps their assignments. It is repeated ρ times.
16. ***Swap ρ customers inter-routes***: Randomly selects two cus-tomers served by the same plant in different vehicle routes in a given period t, and swaps their assignments. The insertion fol-lows the cheapest rule. It is repeated ρ times.
17. ***Swap ρ customers intra-plants***: Randomly selects two cus-tomers served by different plants in a given period, and swaps their assignments, following the cheapest insertion rule. It is re-peated ρ times.

5.2. MIP subproblems

Our matheuristic scheme exploits the solutions obtained from three MIP subproblems. These MIPs provide an initial solution for the 2E-IRPFM, which is polished and improved by our ALNS opera-tors, by a solution improvement procedure, and finally by a general route improvement procedure.

5.2.1. Initial solution procedure

The initial solution procedure (ISP) simplifies the 2E-IRPFM model, disregarding route decisions. To this end, all deliveries are scheduled based on direct links between plants and customers, while minimizing pickups, inventory holding, and fleet manage-ment costs. After we identify all customers served for each vehicle in each period t, the visiting sequence is determined as a traveling salesman problem (TSP). We employ the branch-and-cut technique proposed by Padberg and Rinaldi (1991). The decision variables are precisely the same of the 2E-IRPFM formulation, except for route variables y. The ISP model is formulated by:

$$\min \sum_{t \in T} \Big(\sum_{j \in P} \sum_{k \in K} f_w W_j^{kt} + \sum_{j} \sum_{k} f_s S_j^{kt} + \sum_{j} h_j I_j^t + \sum_{l} h_l I_l^t + \sum_{k \in K} \sum_{(i,j) \in E} 2 c_{ij} X_{ij}^{kt} + \sum_{j \in P} \sum_{k \in K} \sum_{l \in C} c_{jl} Y_{jl}^{kt} \Big) \tag{62}$$

subject to (2)–(12), (15)–(40) and to:

$$Y_{jj}^{kt} \leq \sum_{l \in C} Y_{jl}^{kt} \quad j \in P, k \in K, t \in T. \tag{63}$$

The objective function (62) minimizes the fleet management (rental and cleaning), inventory holding, pickup and approximated delivery transportation costs, calculated as a direct link cost c_{jl}, be-tween customer l and plant j. Constraints (63) ensure that each ve-hicle rented by a given plant performs a delivery route if at least one customer is served in this period. Valid inequalities (47)–(54) and (57)–(60) also apply and the OU policy is handled through constraints (43). After solving the ISP, the TSP B&C Padberg and Ri-naldi (1991) is solved for each vehicle used in the solution, which already respects capacity due to constraints (11).

5.2.2. Solution improvement procedure

The solution improvement procedure (SIP) plays a central role in the optimization process. It first completes an 2E-IRPFM solu-tion, after each ALNS iteration, determining fleet assignment, pick-ups, and delivery quantities. It can also slightly improve a solution by removing or inserting delivery customers, scheduling pickups, and swapping customers among routes. Since a destroy ALNS op-erator may yield an infeasible solution, the insertion mechanism embedded in our SIP can recover feasibility. The parameters used are presented as follows.

- a^{kt}_{jl} : routing reduction cost if customer l is removed from ve-hicle k rented by plant j in period t, where $Y_{jl}^{kt} = 1$. This cost follows the cheapest removal rule.
- b^{kt}_{jl} : routing cost if customer l is inserted in the route of vehicle k rented by plant j in period t, where $Y_{jl}^{kt} = 0$. This cost follows the cheapest insertion rule.
- ψ^{kt}_{jl} : a binary parameter equal to 1, if customer l is served in the current route of vehicle k rented by plant j in period t, where $Y_{jl}^{kt} = 1$, 0 otherwise.

Our SIP model keeps all decision variables from the 2E-IRPFM, except for the y routing variables. Also, visiting customer variables Y are replaced by the following new ones:

- $\delta^{kt}_{jl} = 1$ if customer l is removed from the existing route of vehi-cle k rented by plant j in period t, which obviously has $\psi^{kt}_{jl} = 1$, and 0 otherwise;
- $\omega^{kt}_{jl} = 1$ if customer l is inserted in the route of vehicle k rented by plant j in period t, which obviously has $\psi^{kt}_{jl} = 0$, and 0 oth-erwise.

The SIP model is then formulated as:

$$\min \sum_{t \in T} \Big(\sum_{j \in P} h_j I_j^t + \sum_{l \in C} h_l I_l^t + \sum_{k \in K} \sum_{(i,j) \in E} 2 c_{ij} X_{ij}^{kt} + \sum_{j \in P} \sum_{k \in K} f_w W_j^{kt} + f_s S_j^{kt} + \sum_{l \in C} b^{kt}_{jl} \omega^{kt}_{jl} - \sum_{l \in C} a^{kt}_{jl} \delta^{kt}_{jl} \Big) \tag{64}$$

subject to (2)–(9), (11), (15)–(39), and to:

$$\omega^{kt}_{jl} \leq 1 - \psi_{jl}^{kt} \quad j \in P, l \in C, k \in K, t \in T \tag{65}$$

$$\delta^{kt}_{jl} \leq \psi_{jl}^{kt} \quad j \in P, l \in C, k \in K, t \in T \tag{66}$$

$$q^{kt}_{jl} \leq \left(\psi_{jl}^{kt} - \delta^{kt}_{jl} + \omega^{kt}_{jl}\right) U_l \quad j \in P, l \in C, k \in K, t \in T \tag{67}$$

$$\sum_{j \in P} \sum_{k \in K} \left(\psi_{jl}^{kt} - \delta^{kt}_{jl} + \omega^{kt}_{jl}\right) \leq 1 \quad l \in C, t \in T \tag{68}$$

$$\psi_{jl}^{kt} - \delta^{kt}_{jl} + \omega^{kt}_{jl} \leq Y_j^{kt}{}_j$$

$$\sum_{j \in P} \sum_{l \in C} \left(\delta^{kt}_{jl} + \omega^{kt}_{jl}\right) \leq G \quad k \in K, t \in T \tag{70}$$

$$\omega^{kt}_{jl}, \delta^{kt}_{jl} \in \{0, 1\} \quad j \in P, l \in C, k \in K, t \in T. \tag{71}$$

The objective function (64) minimizes the inventory holding, pickups, fleet management, removal and insertion costs. Con-straints (65) forbid inserting a customer on a route that already serves it, while (66) guarantee that a customer can only be re-moved if it is served by the route. Constraints (67) link re-moval and insertion variables with quantities delivered. Constraints (68) avoid split deliveries, while constraints (69) enforce the de-parture of a given vehicle k rented by plant j, if any customers are assigned to it. Constraints (70) limit the total number of removals and insertions by a constant $G \in \mathbb{Z}^+$. This condition is valid only for deliveries, while pickups are free to be optimized. This is less strict than the one used by Guimarães et al. (2019). New variables domain are defined by (71).

The performance of SIP is strongly dependent on G. When G is small, feasibility may not be recovered when a destroy operator is applied on the ALNS stage. Otherwise, when its value is large, the removals and/or insertions on the routes may not lead to any improvement, because of the approximated transportation costs. We consider a dynamic arrangement to set up an appropriate value for G, detailed in Section 5.3. The OU policy is handled through the constraints (72).

$$q_{jl}^{kt} \geq \psi_{jl}^{kt} - o_{jl}^{kt} + \omega_{jl}^{kt}\ U_l - I_l^{t-1} \qquad j \in P, l \in C, k \in K, t \in T. \tag{72}$$

As a final polishing, each time the SIP model is solved, the TSP procedure is applied to provide an optimal sequence of visits for each vehicle route, allowing us to compute the cost of a new solution exactly.

5.2.3. General improvement routing procedure

To overcome the drawback arising from the approximated transportation costs present in many ALNS operators, we introduce a novel way to optimize inventory holding and fleet management costs, while performing routing improvements considering real transportation costs. This new approach can also be useful for other routing problems.

The general improvement routing procedure (GIRP) can apply any movement in all existing routes in a given solution (either a removal, an insertion, or a swap), taking into account the true transportation costs, unlike SIP. Let A_{jkt} be the set of customers $l \in C$ served by vehicle k rented by plant j in period t, which obviously has $Y_{jj}^{kt} = 1$. If $A_{jkt} = \emptyset$, it means that there are no customers being served by that vehicle, i.e., $Y_{jj}^{kt} = 0$. Consider also a new set of arcs $E_{jkt} \subseteq E$, with $E_{jkt} = \{(u, v) : u, v \in \{j\} \cup A_{jkt} \cup A^{c}_{jkt}\}$, where E_{jkt} represents edges adjacent to plant j, the set of customers in the existing routes A_{jkt}, and its complement A^{c}_{jkt}, where $|A_{jkt}| > 0$. We note that the set generated by $A_{jkt} \cup A^{c}_{jkt} = C$.

Since all the decisions concerning fleet management, inventory flows, pickup, and deliveries are interdependent, GIRP works as the 2E-IRPFM model, with a much smaller search space. In this sense, removals, insertions, and customers swaps are allowed only on established routes, avoiding customers from being served by new ones. Formally, variables q_{jl}^{kt}, Y_{jl}^{kt} and y_{uv}^{kjt} are free to be optimized if and only if their associated routes exist, where $(u, v) \in E_{jkt}$ and $|A_{jkt}| > 0$. Otherwise, q_{jl}^{kt}, Y_{jl}^{kt} and y_{uv} are set to zero, where $(u, v) \in E_{jkt}$ and when $A_{jkt} = \emptyset$.

The remainder of the 2E-IRPFM formulation is not affected, and all other decision variables are free to be optimized. Moreover, for a given solution and a positive integer parameter B, we add the following constraints to the GIRP model, inspired by the local branching constraints of Fischetti and Lodi (2003).

$$\sum_{l \in A_{jkt}} \left(1 - Y_{jl}^{kt}\right) + \sum_{l \in A^{c}_{jkt}} Y_{jl}^{kt} \leq B \qquad j \in P, k \in K, t \in T, |A_{jkt}| > 0. \tag{73}$$

The left hand side of constraints (73) counts the number of binary variables exchanging their value with respect to each existing route from a solution $\bar{s}$, either from 1 to 0 or from 0 to 1, respec-

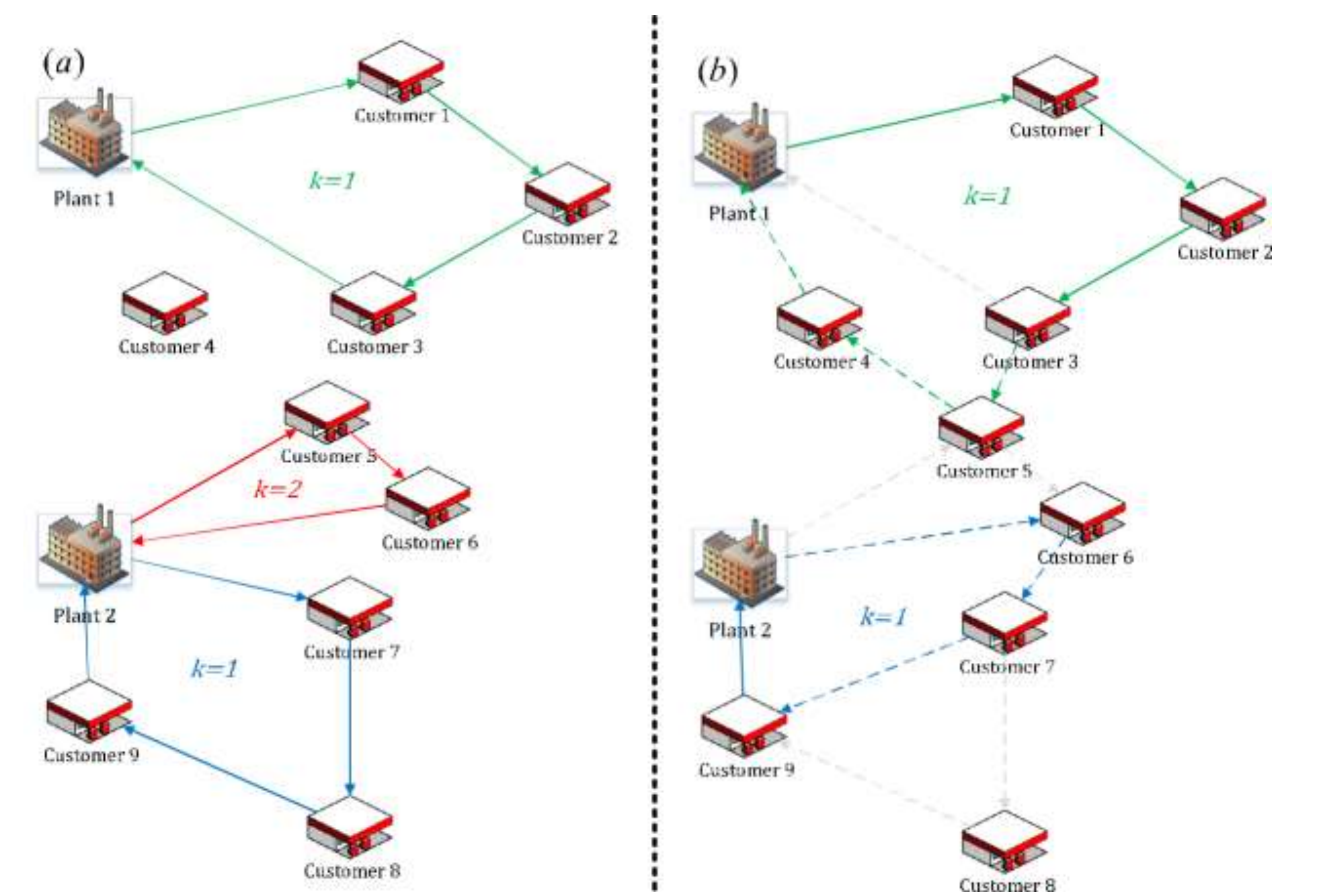

Fig. 2. An example for the general improvement routing procedure (GIRP), before (a) and after (b) all movements.

Table 1

Values of all parameters of the Math-ALNS.

Parameter	Value
Number of iterations	1000
T_{start}	8000
φ	0.989
η	0.8
	20
σ_1	10
σ_2	5
σ_3	2
ρ	5, if $\|C\| > 5$
	2, if $\|C\| = 5$

tively. The set of 2E-IRPFM solutions satisfying (73) define the B-OPT neighborhood $N(\bar{s}, B)$ of $\bar{s}$. The neighborhood size B should be properly chosen, so that the neighborhood $N(\bar{s}, B)$ must be small enough to be explored thoroughly in a reasonable time, but suffi-ciently large to maximize the probability of finding solutions better than $\bar{s}$. Beyond that, we note that all valid inequalities (44)–(60) are applicable to GIRP and the OU inventory policy is ensured by imposing constraints (43).

Fig. 2 shows an example of how the GIRP works. Fig. 2a shows a solution for which the sets $A_{11t} = \{1, 2, 3\}$, $A_{21t} = \{7, 8, 9\}$ and $A_{22t} = \{5, 6\}$ represent all customers served by vehicle $k = 1$ rented by plant $j = 1$, and $k = 1$ and $k = 2$ from plant $j = 2$, re-spectively. Setting $B = 2$, GIRP is able to perform up to two move-ments on each route. As shown in Fig. 2b, unserved customer $l = 4$, and also $l = 5$ who was served by $k = 2$ from $j = 2$, are inserted on route $k = 1$ from plant $j = 1$. This last insertion removes $l = 5$ from $k = 2$. As customer $l = 6$ is inserted in route $k = 1$, these two re-movals empty vehicle $k = 2$, while the removal of $l = 8$ leads to one insertion and one removal in vehicle $k = 1$ from plant $j = 2$. Finally, $A_{11t} = \{1, 2, 3, 4, 5\}$ and $A_{21t} = \{6, 7, 9\}$ are the new sets of customers served in t, while all others $A_{jkt} = \emptyset$.

5.3. Math-ALNS general framework

Our Math-ALNS starts by solving the ISP model and all asso-ciated TSPs, which provide an initial solution s_{ini}. When an ALNS operator is applied on a given solution s (s_{ini} in the first iteration), a neighboring solution s is obtained by solving the SIP model. We initially define $G = n + m$ where $n = |P|$ and $m = |C|$, which guar-antees feasibility is recovered if it was lost by a destroy operator.

While $z(s) < z(s)$, G is decreased by one unit until $G = 1$, and the SIP model is solved after each ALNS iteration. Otherwise, if $z(s) \le (1 + \ )z(s)$, where $\sim U[0.05, 0.15]$, we accept s as a new incumbent solution. Then, we also define $G = max$ $\xi * (n + m), 1$, with $\xi \sim U[0.1, 0.2]$ and solve SIP once again. After that, all indi-vidual routes are optimized by solving their associated TSPs.

Whenever a new best solution s_{best} is found, we enumerate all its routes. Then, the GIRP model is generated and solved, yield-ing s . Due to the critically of neighborhood-size B, we only ac-cept the neighbor solution if $z(s) < z(s_{best})$, and then the opti-mization process continues. Because of the complexity of SIP and GIRP subproblems, our Math-ALNS is executed for about 1000 iter-ations. In order to roughly generate this number of iterations, we set $T_{start} = 8000$ and $\varphi = 0.989$. Scores are updated with $\sigma_1 = 10$, $\sigma_2 = 5$ and $\sigma_3 = 2$, and the reaction factor η is set to 0.8. We de-fine = 20, when weights are updated, scores are set to zero, and the value of is redrawn. Table 1 shows the value of all parame-ters of the Math-ALNS. A pseudocode of our matheuristic is pro-vided in Algorithm 2.

Algorithm 2 Math-ALNS pseudocode.

1: Initialize weights to 1 and scores to 0 for all operators; ~//ALNS Initialization
2: Solve ISP and all associated TSPs, yielding s_{ini}. ~//initial solution
3: $s_{best} \leftarrow s \leftarrow s_{ini}$; $T \leftarrow T_{start}$; $iter = 0$; Generate ;~//parameter settings
4: **while** $T \ge 0.01$ and $iter \le 1,000$ **do**
5: Select and apply an operator i to s; set $G = n + m$; ~//routing perturbation
6: Solve SIP and all associated TSPs, yielding s ; ~//solution reconstruction
7: **if** $z(s) < z(s)$ **then**
8: $s \leftarrow s$; ~//current solution assignment
9: Set $G = max\{G - 1, 1\}$; ~//perturbation polishing
10: Solve SIP and all associated TSPs, yielding s ; ~//solution polishing
11: **if** $z(s) < z(s)$ **then**
12: $s \leftarrow s$ and go to step (9); ~//new current solution assignment
13: **else**
14: **if** $z(s) \le (1 + \ )z(s)$ **then**
15: Set $G = max\{\xi * (n + m), 1\}$ and go to step (10); ~//perturbation polishing
16: **end if**
17: **end if**
18: **if** $z(s) < z(s_{best})$ **then**
19: $s_{best} \leftarrow s$; ~//best solution assignment
20: $\pi_i \leftarrow \pi_i + \sigma_1$; ~//ALNS score operators update
21: Solve GIRP, yielding s ; ~//general solution improvement
22: **if** $z(s) < z(s_{best})$ **then**
23: $s_{best} \leftarrow s \leftarrow s$; ~//best solution assignment
24: **end if**
25: Go to step (9); ~//best solution polishing
26: **else**
27: $\pi_i \leftarrow \pi_i + \sigma_2$; ~//ALNS score operators update
28: **if** $z(s) \le (1 + \ )z(s_{best})$ **then**
29: Set $G = max\{\xi * (n + m), 1\}$ and go to step (10); ~//perturbation polishing
30: **end if**
31: **end if**
32: **else**
33: **if** $e^{(z(s)-z(s))/T} >$, where $\sim U[0, 1]$ **then**
34: $s \leftarrow s$; ~//current solution assignment by simulated annealing
35: $\pi_i \leftarrow \pi_i + \sigma_3$; ~//ALNS score operators update
36: **end if**
37: **end if**
38: **if** ($iter$ mod $= 0$ **then**
39: Generate , update the weights of all operators and reset their scores; ~//parameter settings
40: $s \leftarrow s_{best}$; ~//current solution assignment
41: **end if**
42: $iter \leftarrow iter + 1$; ~//iteration update
43: $T \leftarrow \varphi T$; ~//simulated annealing temperature update
44: **end while**
45: **return** s_{best} ;

6. Branch-and-cut-based ALNS algorithm

During preliminary experiments, we observed that our B&C al-gorithm is very effective in solving small size instances, but as ex-pected from an exact procedure, the upper bound quality deteri-orates dramatically for medium and large instances. On the other hand, our Math-ALNS is very powerful to find good upper bounds in very short computing, even for very large instances. Based on that, we hybridize a branch-and-cut-based ALNS scheme, which we

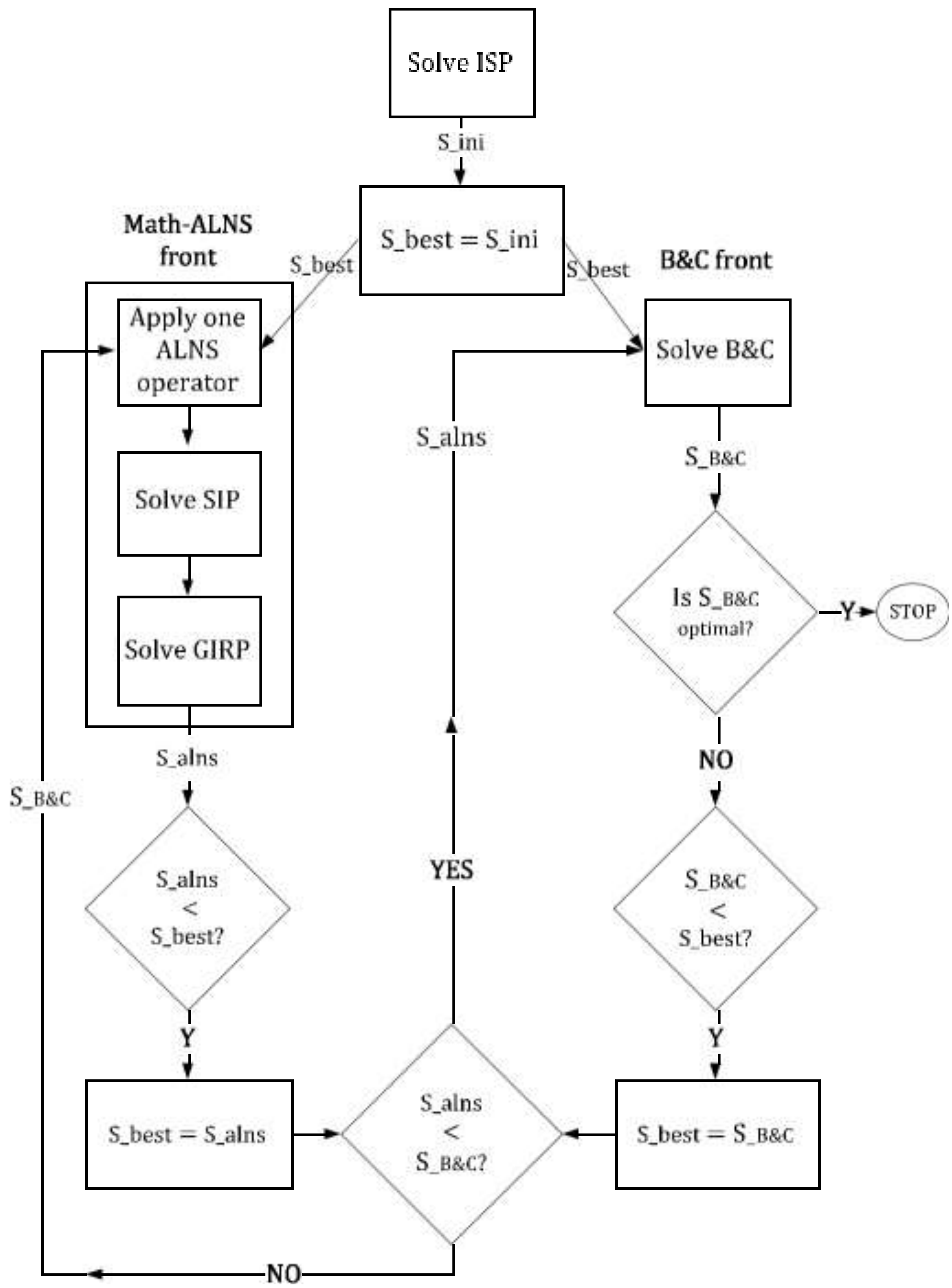

Fig. 3. Scheme of our H-B&C parallel algorithm.

name H-B&C, in order to allow B&C algorithm to take advantage of the good solutions of our Math-ALNS and vice-versa.

Employing parallel computing, H-B&C starts from two fronts, one solving the pure B&C of Section 4, and the other one exe-cuting our Math-ALNS described in Algorithm 2. Whenever a new best solution is found by one of the algorithms, it is immediately provided to other one. This strategy is used to provide better up-per bounds to B&C, especially in large instances. Since B&C stops when the optimal solution is found, it avoids wasting time explor-ing ALNS neighborhoods over and over again on small instances. The algorithm runs until a time limit is reached or an optimal so-lution is proved, which characterizes it as an exact method. Fig. 3 illustrates the dynamics of the proposed H-B&C algorithm.

7. Computational experiments

All algorithms were coded in C++, executed on a grid of Intel(R) Xeon(R) processors at 2.60GHz with up to 16 GB of RAM per node, running in CentOS Linux operating system. All MIPs were solved by Gurobi 8.1.0 and both pure B&C and Math-ALNS were processed using six threads. After preliminary tests, we split the hybrid al-gorithm H-B&C in four threads dedicated to B&C front, and two

Table 2
Comparison between B&C and H-B&C on the 2E-IRPFM.

			ML							OU						
|C|	|P|	|F|	B&C				H-B&C			B&C				H-B&C		
			OPT	SF	$\overline{LB}$	$\overline{T\ (s)}$	OPT	$\overline{LB}$	$\overline{T\ (s)}$	OPT	SF	$\overline{LB}$	$\overline{T\ (s)}$	OPT	$\overline{LB}$	$\overline{T\ (s)}$
5	1	1	16	16	4423.4	0.9	16	4423.4	3.4	16	16	4672.8	1.6	16	4672.8	4.7
	2	2	16	16	4383.5	375.2	16	4383.5	313.1	16	16	4719.0	363.6	16	4719.0	210.9
	2	3	16	16	4343.1	225.7	16	4343.1	417.2	16	16	4679.7	232.1	16	4679.7	233.3
	3	2	14	16	4151.9	1628.4	16	4191.3	1359.1	15	16	4464.8	1276.4	16	4495.1	1024.2
10	1	1	16	16	6128.4	5.9	16	6128.4	10.0	16	16	6460.1	12.8	16	6460.1	23.6
	2	2	8	16	5959.7	3823.2	8	5901.6	3731.6	8	16	6268.0	3812.1	8	6237.0	3857.2
	2	3	8	16	5942.3	3689.3	8	5861.1	3712.1	8	16	6202.9	3773.0	8	6192.5	3814.9
	3	2	8	16	5646.0	3997.0	8	5650.8	4044.8	8	16	5966.7	3733.5	8	5989.5	3693.4
25	1	1	16	16	7898.8	128.9	16	7898.8	144.2	16	16	8635.7	479.2	16	8635.7	633.7
	2	2	8	16	6932.5	4613.3	8	6962.5	4817.4	3	16	7433.2	6803.7	2	7469.8	6966.9
	2	3	8	16	6940.5	4422.6	8	6938.3	4678.1	3	16	7425.5	6668.5	3	7474.6	6816.3
	3	2	4	13	6909.5	6536.3	5	6913.1	6534.9	0	13	7368.5	7200.0	0	7404.9	7200.0
50	1	1	7	16	13239.4	5271.1	7	13317.1	5469.1	0	16	13915.2	7200.0	0	14305.4	7200.0
	2	2	0	8	10923.4	7200.0	0	10964.5	7200.0	0	11	11549.5	7200.0	0	11750.3	7200.0
	2	3	0	8	10771.6	7200.0	0	10921.2	7200.0	0	**9**	11455.2	7200.0	0	11664.6	7200.0
	3	2	0	8	10891.9	7200.0	0	10928.2	7200.0	0	6	11886.8	7200.0	0	11978.0	7200.0
Total			**145**	**229**			**148**			**125**	**231**			**125**		
Avg					**7217.9**	**3519.9**		**7232.9**	**3552.2**			**7694.0**	**3947.3**		**7758.1**	**3954.9**

threads to Math-ALNS front. ISP and SIP are solved to optimality while GIRP is executed for 200s. The algorithms ran up to 7200s on each experiment.

7.1. Overall results for 2E-IRPFM

We have adapted the 2E-MDIRP instances proposed by Guimarães et al. (2019), which were derived from Archetti et al. (2007). Four settings are considered: one supplier-one plant, two suppliers-two plants, two suppliers-three plants, and three suppliers-two plants. The number of customers ranges from 5 to 50. Inventory costs and planning horizon make up four groups: low inventory cost with three (absH3low) and six (absH6low) periods, and high inventory cost with three (absH3high) and six (absH6high) periods. To generate a vari-ety of cleaning and rental costs scenarios, we created four ad-ditional groups: low rental-low cleaning (LRLS), low rental-high cleaning (LRHS), high rental-low cleaning (HRLS), and high rental-high cleaning (HRHS) costs. We generate a total of 256 in-stances. Due to the 2E-IRPFM multi-vehicle topology, we con-sider only the three-vehicle instances from the 2E-MDIRP, which are available for each plant at the rental company. The pa-rameters are calculated as follows and all instances and de-tailed results are available from https://www.leandro-coelho.com/ two-echelon-inventory-routing-problem-with-fleet-management/.

- High rental cost (HR): f_W = Q , where ~ $U[0.4, 0.6]$
- Low rental cost (LR): f_W = Q , where ~ $U[0.2, 0.3]$
- High cleaning cost (HS): f_S = Q , where ~ $U[0.3, 0.5]$
- Low cleaning cost (LS): f_S = Q , where ~ $U[0.1, 0.2]$

We start our analysis by presenting the results obtained with the exact algorithms. Table 2 shows a comparison between B&C and H-B&C for ML and the OU policies. On each supply chain con-sidered, we have 16 instances, with four (one instance absH3low, one absH6low, one absH3high, and one absH6high) on each fleet management cost combination (LRLS, LRHS, HRLS, HRHS). The first three columns describe the supply chain structure, where |C|, |P| and |F| represent the number of customers, plants and suppli-ers, respectively. For each method considered, columns **OPT** and **SF** show the number of optimal and feasible solutions found. Column \boldmath *LB* presents the average lower bound, while \boldmath *T (s)* shows the run time.

Table 3
Average results for B&C and H-B&C to 2E-IRPFM, where B&C found a solution.

|C|	SF	B&C			H-B&C		
		$\overline{Z}$	$\overline{GAP}$	$\overline{T\ (s)}$	$\overline{Z}$	$\overline{GAP}$	$\overline{T\ (s)}$
5	128	4488.5	0.2	513.0	4488.5	0.0	445.7
10	128	6520.2	5.3	2855.8	6476.8	5.1	2860.9
25	122	9138.3	12.5	4587.4	8253.3	8.2	4704.7
50	82	12092.9	20.3	6958.9	10583.1	8.0	6983.6
Avg		**8060.0**	**9.6**	**3728.8**	**7450.4**	**5.3**	**3748.8**

Due to the difficulty of the 2E-IRPFM, the performance of the exact methods clearly deteriorate as the supply chain structure be-comes more complex, especially for the OU policy. Given that the H-B&C gets an initial solution from the Math-ALNS front, the algo-rithm finds a solution for every instance. Furthermore, H-B&C was able to find three additional optimal solutions for the ML policy. With the results of this table, we show that while only B&C can be too time consuming for larger instances and even fails to find feasible solutions for some of them, the Math-ALNS always finds feasible solutions and can obtain good solutions even for very large instances. Overall, we show that their combination do not compro-mise the LB quality, showing the efficiency of the H-B&C. Indeed, the hybrid method dominates B&C, as among the 512 instances evaluated on both policies, H-B&C found 212 better solutions than B&C and there were 300 ties.

We also compare the performance of B&C with H-B&C, on the subset of instances where B&C obtained a solution. Table 3 has up to 32 instances on each supply chain structure, making up 128 in-stances per row (64 ML and 64 OU). As observed, H-B&C yielded tighter bounds, in an equivalent running time.

Table 4 presents the mean results among 64 instances on each inventory policy, for Math-ALNS and H-B&C. As H-B&C took advan-tage from the B&C front, it was able to solve small instances in much shorter running time. The UBs were equivalent, showing the quality of our heuristic method. Considering both inventory poli-cies, we highlight that H-B&C found better solutions in 54 cases, against 45 from Math-ALNS. In another perspective, H-B&C reached the best known solution (BKS) in 467 cases while Math-ALNS did in 458, having 51 exclusive BKS, six more than Math-ALNS.

Table 4
Comparison between Math-ALNS and H-B&C on the 2E-IRPFM.

$\|C\|$	ML				OU			
	Math - ALNS		H-B&C		Math - ALNS		H-B&C	
	$\overline{Z}$	$\overline{T(s)}$	$\overline{Z}$	$\overline{T(s)}$	$\overline{Z}$	$\overline{T(s)}$	$\overline{Z}$	$\overline{T(s)}$
5	4335.4	2144.6	4335.4	523.2	4641.6	1807.3	4641.6	368.3
10	6262.2	2771.7	6263.2	2874.6	6690.3	2860.6	6690.5	2847.2
25	8081.4	2873.6	8079.0	4043.6	8852.1	3108.1	8860.1	5404.2
50	12861.3	4388.6	12882.8	6767.3	14289.9	4674.3	14298.4	7200.0
Avg	**7885.1**	**3044.6**	**7890.1**	**3552.2**	**8618.5**	**3112.6**	**8622.7**	**3954.9**

Table 5
Comparison for inventory policies on BKS of the 2E-IRPFM.

$\|C\|$	ML	OU	GAP %
5	4335.4	4641.6	7.1
10	6262.2	6688.6	6.8
15	8070.0	8846.2	9.6
50	12851.114257.311.0		
Avg7879.78608.58.6			

Since H-B&C and Math-ALNS reached similar results without a clear dominance, all following analyses are performed taking into account the BKS on each instance on each inventory policy.

7.2. Cost analysis for 2E-IRPFM

We evaluate the impact of choosing each of the inventory poli-cies from the perspective of total cost. As shown in Table 5, impos-ing the OU policy at customers increases the total cost in almost 8.6% on average, and its impact is greater in more complex sup-ply chain structures. These results are consistent with the findings of Archetti et al. (2007) for the basic IRP, Coelho et al. (2012a) for multi-vehicle IRP, Coelho et al. (2012b) for the IRP with transship-ment, and Guimarães et al. (2019) for the 2E-MDIRP.

Tables 6 and 7 show the portion of the rental and cleaning costs with respect to the total costs, allowing to evaluate the ef-fect of supply chain complexity on average costs, considering dif-ferent costs combinations (see Section 7.1). For the ML policy, it is interesting to observe that in a more complex system with more plants and suppliers, cleaning costs are less representative. For in-stances with 50 customers, one plant and one supplier, the average cleaning costs ranges from 7.9% to 19.5% of the total costs, while for more complex supply chains with 3 plants and 2 suppliers the cleaning costs proportion is no greater than 5.3%. This is due to the fact that deliveries can be better coordinated among different plants, reducing the need for vehicle cleaning. A similar standard can be observed for the OU policy on Table 7. By comparing both tables, another interesting point is that the share of rental costs does not change among the policies, while cleaning costs are not influenced by the complexity of the system. This fact can be ex-plained by the loss of delivery flexibility when the inventory policy is more strict.

Table 8 shows a sensitivity analysis when the rental and clean-ing costs change, with respect to the LRLS case. The total cost in-creases 4.5% (ML) and 4.9% (OU) on average, when the cleaning cost changes from low to high. Due to the higher coordination, it is relevant mentioning that the system complexity can mitigate this variation, especially when instances are large. For both policies, the total cost increases around 12% when rental costs shift from low to high, and more than 18% when both rental and cleaning costs are high.

7.3. Fleet management analysis

We start our analysis by evaluating the impact of the fleet management decisions. In order to yield comparable data results, we have considered the set of 512 instances of the 2E-IRPFM, by fixing the fleet $|K| = 3$ on each plant in $t = 1$, and also avoid-ing returns over the planning horizon, according to the instances adapted from Guimarães et al. (2019). As the variables $W_j^{k,t}$ are all fixed to one, while $R_j^{k,t}$ are set to zero, $\forall t \in T$, these require-ments impose a fixed cost, given by the rental cost times three (the size of fleet) on each plant, along the planning horizon. Its important to highlight that the only decision that can be taken

Table 6
Fleet management cost as % of total cost for the ML policy.

$\|C\|$	$\|P\|$	$\|F\|$	Rental Cost %				Cleaning Cost %			
			LRLS	LRHS	HRLS	HRHS	LRLS	LRHS	HRLS	HRHS
5	1	1	5.2	7.2	10.7	10.1	3.4	3.3	3.1	6.6
	2	2	6.0	7.0	11.1	11.5	1.6	2.8	1.5	3.4
	2	3	6.1	6.5	10.6	11.2	2.1	3.5	2.0	3.3
	3	2	6.5	8.7	13.1	12.6	4.0	3.6	3.7	6.2
10	1	1	10.2	12.0	19.0	19.2	3.1	4.6	2.7	5.3
	2	2	8.9	10.7	15.8	15.3	2.4	2.6	2.6	4.5
	2	3	9.0	11.0	16.1	15.6	2.4	2.6	2.8	4.0
	3	2	8.6	10.1	15.6	15.5	2.8	3.1	3.0	3.6
25	1	1	13.9	15.7	24.9	22.4	7.8	7.9	6.8	11.7
	2	2	12.7	19.9	21.9	20.4	6.4	2.9	6.1	7.5
	2	3	13.8	17.3	22.4	19.6	4.7	4.7	6.1	6.9
	3	2	12.6	16.9	21.3	21.5	4.9	4.6	5.2	6.7
50	1	1	19.9	25.1	30.7	26.3	9.2	10.5	7.9	19.5
	2	2	19.5	21.1	31.7	30.5	6.3	6.2	5.3	6.4
	2	3	19.2	19.7	33.2	31.9	5.5	7.0	6.0	6.7
	3	2	19.6	20.2	32.7	31.7	2.9	4.9	2.5	5.3
	Avg		**12.0**	**14.3**	**20.7**	**19.7**	**4.3**	**4.7**	**4.2**	**6.7**

Table 7
Fleet management cost as % of total cost for the OU policy.

|C|	|P|	|F|	Rental Cost % LRLS	LRHS	HRLS	HRHS	Cleaning Cost % LRLS	LRHS	HRLS	HRHS
5	1	1	6.0	7.7	12.4	11.9	2.1	3.2	2.0	5.8
	2	2	5.8	7.9	10.1	11.5	2.7	2.8	2.6	4.4
	2	3	6.5	7.6	9.7	11.1	2.0	3.2	2.7	3.6
	3	2	7.1	10.1	13.5	12.1	3.7	3.4	3.2	6.1
10	1	1	9.7	11.3	18.2	18.3	3.2	5.2	2.9	6.5
	2	2	8.7	10.6	15.5	15.6	2.7	3.0	3.0	4.6
	2	3	9.0	10.4	16.4	16.4	3.4	2.8	3.1	4.7
	3	2	8.7	11.1	15.6	15.6	3.5	3.3	3.1	6.2
25	1	1	14.8	17.0	24.4	22.4	5.7	7.2	4.5	12.4
	2	2	11.9	16.6	22.0	21.8	6.6	5.6	5.3	7.1
	2	3	12.5	14.9	21.6	20.4	6.2	4.8	5.3	8.0
	3	2	12.7	15.9	22.5	21.9	5.4	4.2	5.2	6.8
50	1	1	22.5	24.6	32.0	29.0	5.6	9.7	4.8	12.2
	2	2	17.8	20.6	29.0	28.3	5.2	6.6	4.7	9.0
	2	3	17.8	21.1	30.4	29.7	6.3	7.1	5.7	8.4
	3	2	18.2	20.5	30.6	29.7	5.1	4.9	4.5	7.8
	Avg		**11.9**	**14.2**	**20.2**	**19.7**	**4.3**	**4.8**	**3.9**	**7.1**

Table 8
Comparison of fleet management costs.

|C|	|P|	|F|	ML LRLS	%HS	%HR	%HRHS	OU LRLS	%HS	%HR	%HRHS
5	1	1	4168.7	3.8	7.2	13.4	4395.2	3.1	8.8	13.4
	2	2	4197.9	2.6	6.1	9.0	4508.8	2.6	6.0	10.0
	2	3	4171.2	2.4	5.7	8.3	4482.2	2.5	5.8	9.3
	3	2	3941.8	3.1	8.9	13.3	4217.0	3.1	9.4	13.9
10	1	1	5693.8	3.9	10.5	16.0	6004.0	4.4	10.0	15.9
	2	2	5857.7	3.3	9.6	13.7	6282.0	3.0	9.2	13.6
	2	3	5841.1	2.6	9.7	13.2	6289.7	2.3	9.4	13.2
	3	2	6105.9	2.8	8.6	11.9	6519.1	3.2	8.6	13.1
25	1	1	6951.7	9.1	16.1	29.3	7654.7	7.1	16.8	27.4
	2	2	7201.8	3.9	13.7	21.0	7892.0	5.9	14.0	21.6
	2	3	7346.2	3.6	11.2	17.7	7941.4	5.1	12.5	18.3
	3	2	7862.4	4.0	12.2	18.3	8600.8	4.0	13.6	18.8
50	1	1	11473.9	13.3	16.3	37.4	12771.7	9.8	18.2	31.3
	2	2	11181.6	5.9	18.8	23.5	12230.5	8.3	18.9	27.2
	2	3	10980.4	4.7	22.6	27.5	12259.0	6.8	19.3	27.1
	3	2	11694.7	2.7	18.7	22.9	13011.3	6.3	17.3	24.5
	Avg		**7166.9**	**4.5**	**12.3**	**18.5**	**7816.2**	**4.9**	**12.4**	**18.7**

Table 9
Performance indices without fleet management.

Average	ML No FM	FM	%	OU No FM	FM	%
Inventory Cost at Plant	63.4	89.5	41.2	62.3	84.3	35.3
Inventory Cost at Customer	410.9	379.4	**-7.7**	696.4	687.8	**-1.2**
Pickup Routing Cost	1063.3	1044.4	**-1.8**	1065.2	1043.2	**-2.1**
Delivery Routing Cost	4197.8	4342.4	3.4	4463.3	4603.1	3.1
Rental Cost	8364	1572.6	**-81.2**	8364	1696.5	**-79.7**
Cleaning Cost	8.1	461.9	5602.5	11.1	507.7	4473.9
Total Cost	14107.5	7890.2	**-44.1**	14662.3	8622.6	**-41.2**
Deliveries	5.3	4.8	**-9.4**	5.7	5.1	**-10.5**
Pickups	1.1	1.1	0.0	1.1	1.1	0.0

by the plants concerns the vehicle cleaning. When the fleet man-agement decisions are taken into account, plants are free to de-cide how many vehicles will be rented, kept and returned, up to the limit of three rentals per period. This strategy allows to inves-tigate the effects of the fleet management decisions in compari-son with a fixed fleet scenario. Table 9 presents the average par-tial costs, number of deliveries and number of pickups for all of the 512 instances, under the ML and OU policy. In the fixed fleet scenario, we observe an increase of 41% on the average inventory cost at the plant for the ML policy, and 35% for the OU policy, when the fleet is fully managed. Since pickup decisions are de-pendent on the availability of vehicles, plants have fewer options to schedule them and tend to maintain higher inventory levels of input. At the same time, the inventory average cost at customers is less representative on the total average cost when the fleet is fixed, which explains the reduction of this partial cost when an outsourced managed fleet is considered. Another point we notice is the average decrease on the number of deliveries by around 10% for both inventory policies. When vehicles are not fully available and need to be rented, deliveries routes tend to be longer and less frequent, which explains the increase around 3% on the average deliveries routing costs. In general, when fleet management is con-

Table 10
Performance indices without fleet management.

	GFU				NFU				q/dist			
$\|C\|$	LRLS	LRHS	HRLS	HRHS	LRLS	LRHS	HRLS	HRHS	LRLS	LRHS	HRLS	HRHS
5	0.12	0.12	0.12	0.12	0.70	0.70	0.70	0.70	0.28	0.28	0.28	0.28
10	0.18	0.18	0.18	0.18	0.77	0.77	0.77	0.77	0.48	0.48	0.48	0.48
25	0.17	0.17	0.17	0.17	0.79	0.76	0.78	0.75	0.76	0.74	0.75	0.74
50	0.17	0.17	0.18	0.17	0.76	0.76	0.77	0.76	1.29	1.25	1.30	1.26
Avg	**0.16**	**0.16**	**0.16**	**0.16**	**0.76**	**0.75**	**0.76**	**0.74**	**0.70**	**0.69**	**0.70**	**0.69**

Table 11
Performance indices with fleet management.

	GFU				NFU				q/dist			
$\|C\|$	LRLS	LRHS	HRLS	HRHS	LRLS	LRHS	HRLS	HRHS	LRLS	LRHS	HRLS	HRHS
5	0.72	0.58	0.75	0.73	0.72	0.72	0.75	0.75	0.28	0.28	0.28	0.28
10	0.80	0.66	0.83	0.81	0.80	0.80	0.83	0.84	0.48	0.48	0.48	0.48
25	0.88	0.67	0.91	0.90	0.88	0.86	0.91	0.91	0.72	0.70	0.71	0.68
50	0.88	0.74	0.88	0.86	0.88	0.87	0.88	0.87	1.27	1.23	1.26	1.19
Avg	**0.82**	**0.66**	**0.84**	**0.83**	**0.82**	**0.81**	**0.84**	**0.84**	**0.69**	**0.68**	**0.68**	**0.66**

Table 12
Average fleet usage.

Rental Cost	$\|C\|$	Low Cleaning Cost				High Cleaning Cost				Comparative Analysis			
		#W	#S	#R	#(X+Y)	#W	#S	#R	#(X+Y)	**% W**	**% S**	**% R**	**% (X+Y)**
Low	5	4.2	2.8	1.7	5.2	5.4	1.3	0.3	5.3	30.5	-51.6	-82.4	0.6
	10	5.7	3.0	2.0	6.7	7.1	1.3	0.3	6.8	23.2	-56.1	-87.5	0.8
	25	5.0	3.5	2.4	6.1	6.7	1.3	0.2	6.2	33.1	-63.7	-91.7	1.8
	50	5.1	2.7	1.6	6.2	6.1	1.3	0.3	6.2	20.3	-51.0	-85.0	0.9
Avg		**5.0**	**3.0**	**1.9**	**6.0**	**6.3**	**1.3**	**0.2**	**6.1**	**26.8**	**-55.6**	**-86.6**	**1.0**
High	5	4.0	2.8	1.8	5.1	4.2	2.1	1.0	5.2	5.3	-27.0	-43.5	1.9
	10	5.5	3.3	2.3	6.5	5.7	2.1	1.1	6.5	4.2	-34.4	-50.9	0.5
	25	4.8	3.7	2.6	5.9	4.9	2.2	1.2	5.9	1.0	-38.0	-52.9	-0.2
	50	5.0	2.8	1.8	6.1	5.1	2.0	0.9	6.1	1.7	-28.9	-42.7	0.0
Avg		**4.8**	**3.2**	**2.1**	**5.9**	**5.0**	**2.1**	**1.1**	**5.9**	**3.0**	**-32.1**	**-47.5**	**0.5**

Table 13
Average results for strategies: without GIRP, $G = 1$, and $G = m + n$.

Strategy	$\bar{Z}$	$\%\bar{Z}$	$\bar{T}$ (s)	$\%\bar{T}$ (s)
Original Math-ALNS	6740.40	-	3042.64	-
Without GIRP	6896.16	1.36	2447.89	-19.54
With $G = 1$ fixed	6815.99	0.67	2293.73	-24.61
With $G = n + m$ fixed	6820.80	0.73	2330.68	-23.39

Table 14
Average results when removing one ALNS operator at a time.

Strategy	$\bar{Z}$	$\%\bar{Z}$
Original Math-ALNS	6740.40	-
Randomly remove ρ	6851.27	0.97
Remove worst ρ	6880.26	1.26
Shaw removal route based	6835.58	0.85
Empty one period	6857.19	1.01
Empty one vehicle	6811.24	0.65
Empty one plant	6861.14	1.07
Farthest customer	6820.22	0.70
Avoid consecutive visits	6855.08	1.04
Remove ρ minimum residual deliveries	6804.29	0.59
Remove ρ most visited customers	6857.80	1.06
Randomly insert ρ	6828.04	0.80
Insert best ρ	6856.93	1.03
Assignment to the nearest plant	6860.57	1.05
Shaw insertion	6796.66	0.45
Swap ρ customers	6841.86	0.92
Swap ρ customers inter-routes	6854.14	1.00
Swap ρ customers intra-plants	6850.74	0.98

sidered, the total average cost decreases by around 40% for both policies.

We also carry out an analysis regarding the performance in-dices. Aiming to provide a compact evaluation, we investigate the quantity-distance (*q*/*dist*) ratio, which computes the volume deliv-ered per distance traveled. Eq. (74) shows the index.

$$\frac{q}{dist} = \frac{\sum_{t\in T}\sum_{j\in P}\sum_{k\in K}\sum_{l\in C} q^{kt}_{jl}}{\sum_{t\in T}\sum_{j\in P}\sum_{k\in K}\sum_{(u,v)\in E} c_{uv} y^{kjt}_{uv}}. \quad (74)$$

As pointed by Song and Savelsbergh (2007), *q*/*dist* is not ef-fective to measure absolute performance. We then introduce two new performance indices, in order to evaluate fleet management strategies among different costs combinations. The gross fleet us-age (*GFU*) shown in (75) computes the average occupancy of the rented fleet. The numerator calculates the total volume delivered to customers, while the denominator computes the total rented ca-pacity. *GFU* is particularly useful to compute rented fleet idleness, derived from delivery and fleet management strategy facing differ-ent rental and cleaning costs combinations.

$$GFU = \frac{\sum_{t\in T}\sum_{j\in P}\sum_{k\in K}\sum_{l\in C} q^{kt}_{jl}}{\sum_{t\in T}\sum_{j\in P}\sum_{k\in K} W_{jkt}\, Q} \quad (75)$$

Given that rented vehicles can be housed at plants and not be-ing used, the net fleet usage (*NFU*) presented in (76) computes the average occupancy of the fleet. Thus, it is possible to split the de-

Table 15
Comparison between B&C from Guimarães et al. (2019), new B&C and H-B&C to 2E-MDIRP, ML policy.

$\|C\|$	$\|P\|$	$\|F\|$	B&C Guimarães et al. (2019)				B&C				H-B&C			
			OPT	SF	$\overline{LB}$	$\overline{T\,(s)}$	OPT	SF	$\overline{LB}$	$\overline{T\,(s)}$	OPT	SF	$\overline{LB}$	$\overline{T\,(s)}$
5	1	1	4	4	3781.5	1.9	4	4	3781.5	1.2	4	4	3781.5	2.7
	2	2	4	4	3822.5	81.9	4	4	3822.6	52.7	4	4	3822.6	278.4
	2	3	4	4	3786.2	66.9	4	4	3786.2	57.4	4	4	3786.2	430.3
	3	2	4	4	3484.6	238.7	4	4	3484.6	478.3	4	4	3484.6	1461.6
10	1	1	4	4	4897.7	54.1	4	4	4897.7	20.5	4	4	4897.7	48.4
	2	2	2	4	4661.8	3716.2	2	4	4695.6	4326.9	2	4	4674.5	3734.9
	2	3	2	4	4702.7	3697.0	2	4	4707.4	3823.5	2	4	4683.8	3644.8
	3	2	2	4	4421.1	3812.5	2	4	4682.6	5286.7	2	4	4606.9	4120.3
25	1	1	2	4	5305.0	4314.5	4	4	5403.4	626.7	4	4	5403.4	838.7
	2	2	0	2	4538.7	7200.0	2	4	4998.0	3862.0	2	4	4951.8	4126.8
	2	3	1	2	4574.6	6920.0	2	4	4962.2	3854.6	2	4	5012.2	3868.1
	3	2	0	2	4399.8	7200.0	2	4	4971.2	5521.2	2	4	4899.8	5065.9
50	1	1	0	2	7365.1	7200.0	0	3	7468.1	7200.0	0	4	7454.8	7200.0
	2	2	0	0	6500.5	7200.0	0	1	6752.0	7200.0	0	4	6815.2	7200.0
	2	3	0	1	6693.7	7200.0	0	2	6810.2	7200.0	0	4	6729.9	7200.0
	3	2	0	0	6143.6	7200.0	0	1	6727.8	7200.0	0	4	6819.6	7200.0
Total			**29**	**45**			**36**	**55**			**36**	**64**		
Avg					**4942.4**	**4131.5**			**5121.9**	**3544.5**			**5114.0**	**3526.3**

Table 16
Comparison between B&C from Guimarães et al. (2019), new B&C and H-B&C to 2E-MDIRP, OU policy.

$\|C\|$	$\|P\|$	$\|F\|$	B&C Guimarães et al. (2019)				B&C				H-B&C			
			OPT	SF	$\overline{LB}$	$\overline{T\,(s)}$	OPT	SF	$\overline{LB}$	$\overline{T\,(s)}$	OPT	SF	$\overline{LB}$	$\overline{T\,(s)}$
5	1	1	4	4	3998.2	2.1	4	4	3998.2	2.3	4	4	3998.2	5.3
	2	2	4	4	4085.4	431.6	4	4	4085.4	416.9	4	4	4085.4	219.6
	2	3	4	4	4065.7	320.4	4	4	4065.7	319.3	4	4	4065.7	189.6
	3	2	4	4	3733.4	1057.0	4	4	3733.3	1261.1	4	4	3733.5	778.9
10	1	1	4	4	5204.2	114.1	4	4	5204.2	33.1	4	4	5204.2	235.2
	2	2	2	4	4799.2	3700.4	2	4	4927.8	4460.6	2	4	4999.8	3746.3
	2	3	2	4	4811.2	3801.3	2	4	4933.4	3715.0	2	4	4920.8	3765.6
	3	2	2	4	4597.9	3647.5	2	4	4817.8	3660.8	2	4	4831.9	3666.6
25	1	1	1	3	5262.3	5663.3	3	4	5800.4	3462.7	2	4	5767.5	4207.6
	2	2	0	2	4664.8	7200.0	1	4	5452.7	6453.8	1	4	5317.2	6637.6
	2	3	0	2	4548.0	7200.0	0	4	5453.1	7200.0	0	4	5408.5	7200.0
	3	2	0	2	4344.7	7200.0	0	4	5320.9	7200.0	1	4	5297.3	6508.6
50	1	1	0	1	7773.6	7200.0	0	3	8016.2	7200.0	0	4	8044.7	7200.0
	2	2	0	0	7034.9	7200.0	0	1	7413.5	7200.0	0	4	7442.8	7200.0
	2	3	0	0	7178.6	7200.0	0	1	7358.2	7200.0	0	4	7359.7	7200.0
	3	2	0	0	6627.3	7200.0	0	2	7489.8	7200.0	0	4	7539.1	7200.0
Total			**27**	**42**			**30**	**55**			**30**	**64**		
Avg					**5170.6**	**4321.1**			**5504.4**	**4186.6**			**5501.0**	**4122.6**

livery strategy from fleet management decisions in our analyses.

$$NFU = \frac{\sum_{t\in T}\sum_{j\in P}\sum_{k\in K}\sum_{l\in C} q_{jl}^{kt}}{\sum_{t\in T}\sum_{j\in P}\sum_{k\in K} Y_j^{kt} Q_j}. \quad (76)$$

In Tables 10 and 11, each row reports the average of 128 instances (64 for each inventory policy), according to the supply chain structure. As expected, the $q/dist$ index does not reveal anything distinctive among cost combinations and fleet management. We observed that for large instances, the fleet carries a greater volume per traveled distance. When the fleet is owned, the *GFU* is 16% in all cost combinations, which shows an inefficient vehicle usage. On the other hand, the occupancy rate measured by *NFU* goes up to 76% for the vehicles used on the deliveries routes. On the fleet management context, the clearest conclusion lies on *GFU* and *NFU* comparison on the LRHS combination. We note that the rented fleet average occupancy is 66%, but for vehicles actually assigned it rises up to 81%. When cleaning cost is low or both are high, there is no remarkable difference between *GFU* and *NFU*. Overall, can use the assets with higher efficiency, leading to cost reduction and better logistics activities coordination.

Finally, Tables 12 clarifies the single fleet management decision as cost parameters change. Column **#W** brings the average of rentals, **#S** the average number of cleanings, **#R** the average return, and **#(X+Y)** the average pickup plus delivery routes. For low rental and low cleaning cost (LRLS) we observe that five rentals, three cleanings, two returns and six routes are performed. When cleaning cost switched to high, it becomes more advantageous to keep the vehicles at the plants, instead of returning them. The number of returns is 86.6% smaller than before, and the number of cleanings is 55.6% smaller as well. When rental costs are high, the cleaning cost is less relevant and the average number of cleanings dropped 32% when cleaning costs changed from low to high. At the same time, we observe a decrease in the number of rentals, especially when cleaning costs are high, decreasing from 6.3 on LRHS to 5.0 on HRHS.

7.4. Performance of the elements of the Math-ALNS

We also perform extensive experiments to evaluate the per-by considering the results presented on Tables 9–11, we can highlight that when fleet management decisions are considered, plants

formance of each element of our Math-ALNS. The experiments were performed with 40 randomly selected instances. Detailed results are available from https://www.leandro-coelho.com/ two-echelon-inventory-routing-problem-with-fleet-management/. Table 13 shows that removing GIRP from Math-ALNS significantly decreases the quality of the results of the algorithm by an average of 1.36%, when compared to the results of the original Math-ALNS. The update procedure of G in SIP is also verified: setting a fixed value $G = 1$ or $G = n + m$ also deteriorates the results by 0.67% and 0.73%, respectively, while the processing time decreases between 20% and 25% in comparison with the original Math-ALNS.

Table 14 evaluates the performance of the algorithm when each ALNS operator is removed. To this end, we have run the algorithm 17 times, each time removing one of the operators. As can be seen in the table, each removal led to a deterioration in the quality of the results. On average, the removal of one operator decreases the quality of the solution by almost 1%. These experiments demon-strate that although the algorithm contains many elements, all of them contribute to the overall results we achieve in this very chal-lenging problem.

7.5. Results for the 2E-MDIRP

We have also compared the performance of our methods on in-stances proposed by Guimarães et al. (2019) for the 2E-MDIRP. We assessed all 64 instances with three vehicles and solved them un-der the ML and the OU policies on the second echelon, making a total of 128 experiments for each algorithm.

The mathematical model proposed in Section 3 is flexible enough to handle all 2E-MDIRP features. To this end, it is sufficient set to zero all of fleet management costs, i.e., $f_S = 0$ and $f_W = 0$. We highlight that all valid inequalities (44)–(60) and the OU pol-icy (43) apply to both problems.

Guimarães et al. (2019) proposed an asymmetric formulation for the 2E-MDIRP, employing symmetry breaking constraints. Our formulation is subtly different, since we consider edge (i, j) only if $i < j$. This reformulation is useful to enable direct deliveries through a single-edge, which substantially reduces the number of decision variables, before the search tree is structured. In addition, we propose a new set of valid inequalities (58)–(60). Therefore, we are able to analyze the performance of the B&C and H-B&C, derived from this new formulation, against the B&C proposed by Guimarães et al. (2019).

Table 15 shows a comparison among exact algorithms for the ML policy. We highlight that our B&C was able to find a solu-tion to all instances up to 25 customers, and 36 optimal solutions were proved, with seven new ones. Our B&C and H-B&C had an equivalent performance in this regard, but H-B&C obtained tighter bounds. Analogously, Table 16 presents the results to OU policy, with similar performance.

To handle large instances, Guimarães et al. (2019) designed a matheuristic under the ALNS mechanism. Although we follow a similar way, our Math-ALNS embeds reformulated MIPs. The main innovation lies in the GIRP model, which has a neighborhood ex-ploration strategy based on real transport costs (see Section 5.2.3). Table 17 compares our Math-ALNS and the matheuristic from Guimarães et al. (2019). Each row shows the average of six-teen instances, four instances (absH3low, absH6low, absH3high, and absH6high) on each supply chain structure (one supplier one plant, two suppliers two plants, two suppliers three plants, and three suppliers two plants). For each inventory policy, columns $\bar{Z}$ Guimarães et al. (2019) and $\bar{T}(s)$ Guimarães et al. (2019) show the average results for total cost and processing time for the matheuristic of Guimarães et al. (2019), while our Math-ALNS is presented in $\bar{Z}$ and $\bar{T}(s)$. The gap between the average total cost is

obtained by $\frac{\bar{Z} - \bar{Z}_{[17]}}{\bar{Z}_{[17]}} \times 100$. As Math-ALNS has been adapted from 2E-IRPFM, it preserves the full set of fleet management constraints,

Table 17
Comparison between the matheuristic of Guimarães et al. (2019) and our Math-ALNS.

$\|C\|$	ML					OU				
5	3718.73	280.77	3718.73	1160.18	0.00	3970.70	313.41	3970.70	874.16	0.00
10	5131.25	874.21	5121.70	2075.28	-0.18	5475.60	1320.02	5474.37	1743.51	-0.02
25	605846.	26902.	615824.	2633.23	-0.36	306429.	301029.	486350.	462457.	-1.19
50	468167.	011246.	618068.	3942.77	-1.19	799105.	232335.	989076.	344486.	-0.32
Avg	**05716.**	**8825.**	**45683.**	**92452.**	**-0.4**	**36245.**	**51249.**	**16218.**	**42390.**	**-0.4**

requiring more time to solve the associated MIPs. Although the overall relative improvement on ***Z*** suggests an equivalent perfor-mance, Math-ALNS was superior especially on large instances. In general, there was improvement in 46 (23 for ML and 23 for OU) out of the 128 instances, with a tie in 81 and only one worse re-sult.

8. Conclusions

In this study, we have introduced the 2E-IRPFM, a new vari-ant of IRP which incorporates fleet planning in a two-echelon lo-gistics system. This problem has a complex many-to-many supply chain structure, where the plants are in the middle layer and must control the inventory and routing decisions regarding input pickup and final product delivery, while managing tactical and operational fleet decisions. We have proposed a MILP formulation and a B&C algorithm to solve the problem, taking into account different in-ventory policies. We have also designed a matheuristic algorithm and an exact hybrid parallel approach to efficiently solve large in-stances. Validation experiments, performed on 2E-MDIRP instances from the literature, showed that our algorithms are very effective, yielding BKSs for the whole set of problems evaluated. We have shown that rentals, cleanings and vehicle returns represent a sig-nificant portion of logistics costs. Moreover, a more complex logis-tics system proved more resilient regarding changing in rental and cleaning costs, leading to greater efficiency in logistics operations.

As future directions, we suggest to tackle production features. The blend process of commercial gasoline requires certain amount of pure gasoline and a complement of ethanol; the decision about when and how much to produce of it enriches even more the problem. Besides that, since the octane grade of commercial gaso-line is defined according to the portion of ethanol on its blend, different products can be considered based on that grade. In this case, a multi compartmentalized fleet can also be considered, in order to transport different products to different customers. More-over, while rich algorithms such as ours allow solving a myriad of intricate problems, the high number of parameters remains an is-sue. Further research can be done in automatically tuning these parameters or designing simpler algorithms capable of achieving of the same quality.

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